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THE VOICE AND EARS OF THE ARMORED FORCES

FEBRUARY 1943

VOLUME 31 NUMBER 2

Wartime Engineering Papers

Medal of Honor, 1943

Voltage-Regulated Power Supplies

Transcription-Turntable Speed Variation

Loop Antennas for Aircraft

Deionization in Harmonic Generator

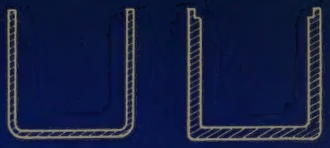
Two-Antenna Array

Lightning Striking Frequencies

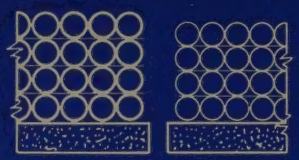
Institute of Radio Engineers

MACHINED ROD TO SPUN TUBING
SAVING 50% BRASS

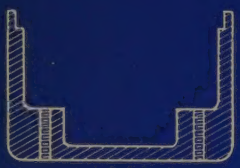
BRASS INSERT AND SCREW TO PK SCREW ONLY
SAVING 100% BRASS

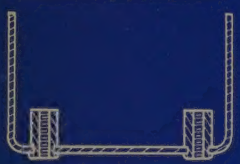


DRAWN ALUMINUM TO BAKELITE
SAVING 100% ALUMINUM

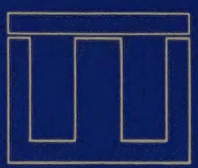


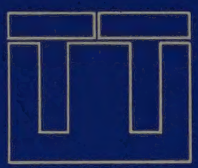
GLASS INSULATED TO FORMVAR
SPACE FACTOR SAVING 10% COPPER



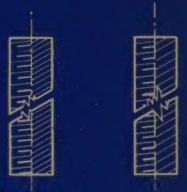


DIECAST TO DRAWN
SAVING 70% ALUMINUM



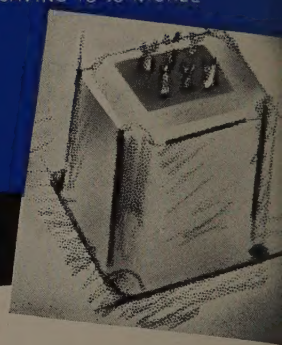


BLANKED LAMINATION TO SCRAPLESS
SAVING 35% SILICON STEEL



STAINLESS TO PLATED STEEL
SAVING 18% NICKEL

"and it can win or lose the war...



We all know that our greatest problem today lies in material shortages. The bulk of this problem . . . and it can win or lose the war . . . lies in our hands. A waste of materials, particularly critical materials, in an engineering design today, is as damnable as sabotage.

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Proceedings *of the* I·R·E

Published Monthly by
The Institute of Radio Engineers, Inc.

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Entered as second-class matter October 26, 1927, at the post office at Menasha, Wisconsin, under the Act of February 28, 1925, embodied in Paragraph 4, Section 538 of the Postal Laws and Regulations. Publication office, 450 Ahnaip Street, Menasha, Wisconsin. Editorial and advertising offices, 330 West 42nd St., New York, N. Y. Subscription $10.00 per year; foreign, $11.00.

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Wartime Radio-and-Electronic Engineering Papers

ENGINEERS in the radio-and-electronic field are vigorously contributing their best efforts toward the ultimate victory of the United Nations. Their efforts are indeed one of the keystones in that eventual triumph. Insofar as their activities are of Military or Naval significance, radio-and-electronic engineers *must* refrain from *any* mention of their work outside of their own restricted and authorized circles.

But some engineers have misinterpreted this correct policy as a reason for preparing *no* papers and publishing *no* information. This attitude is inappropriate and believed not to be in the best national interest. Engineers should not hesitate to prepare and present for publication papers containing no information of military significance to the enemy. From their work in the radio-and-electronic field, it is thought engineers can draw many data, descriptions of methods, or information on nonmilitary instruments which will be of help to their fellow engineers but of no military significance to the enemy.

The Institute of Radio Engineers is highly privileged to quote the following communication from an eminent military and technical leader, Major General Dawson Olmstead, Chief Signal Officer of the Army of the United States of America. General Olmstead states:

"It is, in my opinion, a vital necessity for American engineers to continue their valued research and development of radio engineering in private industry, and to disseminate information that would be helpful to fellow technicians but which would be of no military value to the enemy. This is quite in line with the entire War Department principle of releasing as much information as is consistent with security. There should be a substantial amount of such informational material unaffected by the restrictions that wartime imposes.

"The facilities of this office will be available to you for the review of manuscript that is doubtful from the point of view of containing information that might prove of value to Axis powers. As you indicate, such papers should not be prepared or published. In those isolated instances where doubt exists as to the advisability of publishing certain articles, you may count on my co-operation in having them properly evaluated.

"You may, therefore, quote me in the words of your letter of December 1 to the effect that 'Engineers in wartime can to advantage find time to prepare technical papers containing information of no military value to the enemy but of assistance to their fellow engineers. The preparation of such papers is regarded as being in the national interest.'

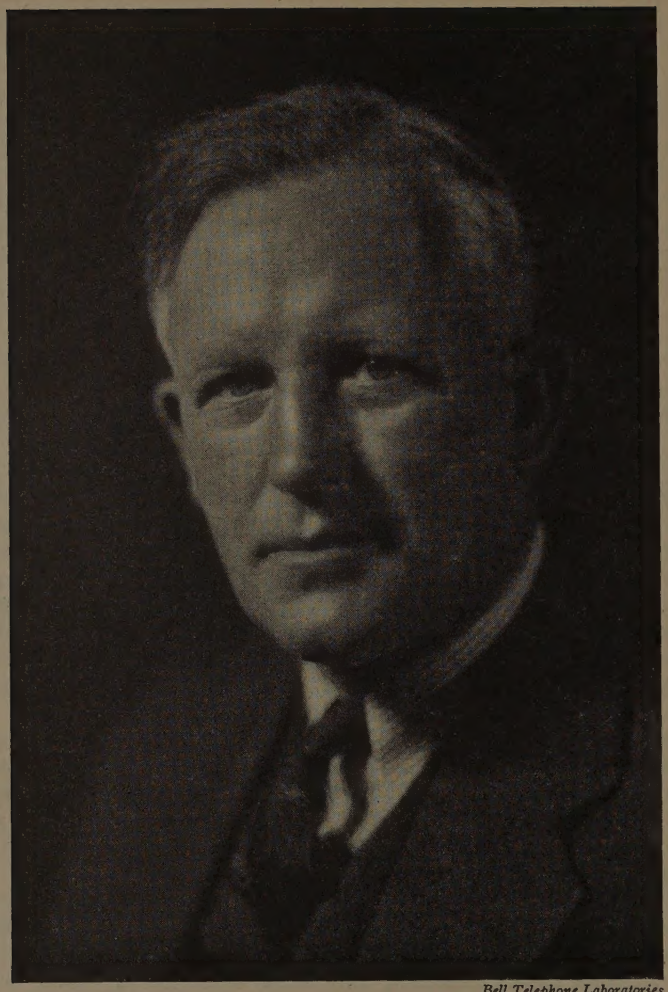
Most sincerely,

(signed) Dawson Olmstead

Dawson Olmstead,
Major General,
Chief Signal Officer of the Army."

Patriotic engineers in the radio-and-electronic field will be grateful to General Olmstead for a clear and constructive definition of policy. They will be guided accordingly. The Institute of Radio Engineers will welcome the papers which they prepare and submit for publication in complete accordance with the policy enunciated above.

The Editor



Bell Telephone Laboratories

WILLIAM WILSON

RECIPIENT, MEDAL OF HONOR, 1943

William Wilson was born at Preston, England, on March 29, 1887. He received the B.Sc. degree in 1907; the M.Sc. degree in 1908; and the D.Sc. degree in 1913 from the University of Manchester (England). From 1907 to 1912 he was occupied in research work on electronic physics at the Universities of Manchester and Cambridge in England and Giessen in Germany. From 1912 to 1914 Dr. Wilson was a lecturer in physics at the University of Toronto. Since 1914 he has been with the Western Electric Company and the Bell Telephone Laboratories. In 1918 he was in charge of the research, development, and manufacture of vacuum tubes; in 1925 he was in charge of radio research and of the development and design of the transatlantic radiotelephone equipment; in 1927 he became Assistant Director of Research, and in 1934 he was placed in charge of research work on problems of wire communication. Late in 1936, Dr. Wilson was appointed Assistant Vice President in charge of the departments of Personnel and Publication.

Dr. Wilson joined the Institute of Radio Engineers as a Member in 1926, transferring to Fellow grade in 1928. He was a member of the Board of Directors of the Institute of Radio Engineers from 1932 to 1936 and served as a member or chairman of numerous Institute committees, Awards, Bibliography, Convention, Nominations, Sections, Papers, Standards, as well as being on its Board of Editors. At the Winter Conference meeting on January 28, 1943, Dr. Wilson was awarded the Medal of Honor for his achievements in the development of modern electronics, including its application to radiotelephony, and for his contributions to the welfare and work of the Institute.

Upon his retirement from the Bell Telephone Laboratories, Dr. Wilson carries with him the hearty appreciation and cordial good wishes of his fellow workers and the members of the I.R.E.

Voltage-Regulated Power Supplies*

Design and Construction Considerations

ALEXANDER B. BERESKIN†, ASSOCIATE, I.R.E.

Summary—*Power supplies with low regulation have always been an important consideration in the electronic and allied fields. This naturally dictates the use of voltage-regulated power supplies of the types described previously by Hunt and Hickman[1] and other investigators.*

It is the purpose of this paper to discuss the problems involved and to develop an orderly procedure for designing and constructing these voltage regulated power supplies for specific applications. The correlation between design data and actual tests on a finished model will also be shown.

Operation of Regulator Circuit

REFERENCE to previous literature will show that many variations of the basic degenerative regulator circuit are available for use but the particular version on which this article is based is shown in Fig. 1.

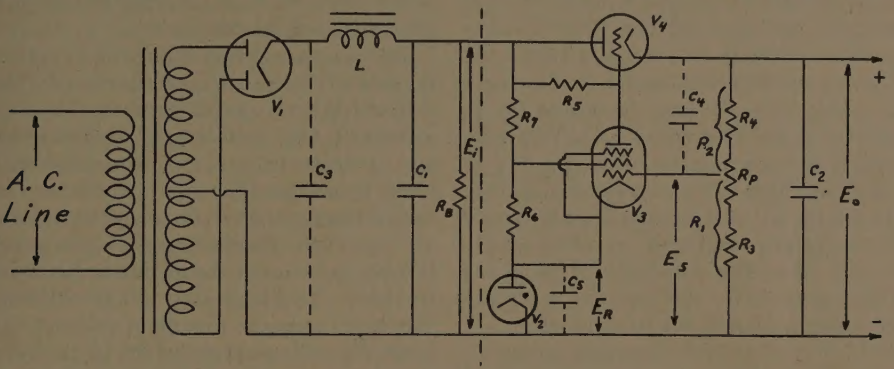


Fig. 1

The section of the diagram, in Fig. 1, shown to the left of the dotted line is a conventional single-phase—full-wave rectifier with choke input to the filter. The condenser C_3 is a condenser of very low capacitance which may be used to minimize "hash" if a gaseous rectifier is used and R_B together with R_7, R_6, and V_2 serves the purpose of bleeding whatever current may be necessary to obtain operation in the "flat" region of the rectifier output characteristic.

The section of the diagram to the right of the dotted line comprises the voltage-regulator portion of the circuit and operates in the manner indicated below. The cathode of the pentode V_3 is kept at a constant potential above the negative side of the line by means of the regulator tube V_2 or any other constant-potential device. If the resistance R_5 is sufficiently large, then the over-all tendency of the regulator circuit is to keep the voltage E_s approximately constant at a value that will keep the grid voltage of V_3 close to the cutoff value. Under this condition the grid current of V_3 is negligibly small and the following relation is established:

$$E_s = \frac{R_1}{R_1 + R_2} E_0.$$

To trace the operation of the circuit, assume first of all that the circuit has been previously balanced and that then, for any reason whatsoever, there is a tendency for E_0 to increase. The increase in E_0 will produce a corresponding increase in E_s and the grid of V_3 will become more positive, thus increasing the plate current in V_3. This increase in current produces a greatly amplified increase in voltage across R_5 and therefore drives the grid of V_4 more negative. This tends to decrease the load current and therefore to bring E_0 back to its original value. If the voltage E_0 had decreased instead, a reverse of the sequence of events mentioned would have taken place.

Design Factors

In order to simplify the discussion of the factors involved in the design of the regulator circuit, it will be broken down into its component parts and each part will be discussed independently.

The fundamental function of the V_2 tube is to maintain a constant potential (E_R) between the cathode of V_3 and the negative side of the line. Since a certain amount of current must be bled from the power supply anyway, it is convenient to use one of the VR75-30, VR105-30, or VR150-30 tubes or a series combination of any number of these tubes depending on the voltage E_R that is required for V_2. Ordinary neon or argon "glow tubes" may also be used although there is usually considerably more variation in their characteristics than in those of the standard voltage-regulator

* Decimal classification: R356.2. Original manuscript received by the Institute, September 24, 1942.

† University of Cincinnati, Cincinnati, Ohio.

[1] F. V. Hunt and R. W. Hickman, *Rev. Sci. Instr.*, vol. 10, p. 6; 1939.

line of tubes. The use of batteries, while perfectly feasible, is usually not convenient due to the necessity of occasional replacement. Under certain conditions oscillations may be started by the regulator tube V_2 but these can usually be stopped by a condenser C_5. This condenser is ordinarily not necessary.

The R_6-R_7 combination of resistors should be designed to limit the current through V_2 to approximately the maximum rated value for the maximum value of E_i that is expected. At the same time, R_6 and R_7 should be proportioned so as to produce the correct value of screen voltage for V_3. In the cases where E_i is liable to vary between wide limits, it may be desirable to use a VR105-30 tube in place of resistor R_6 so as to maintain a constant screen voltage on V_3.

The tube V_3 and the resistor R_5 form a simple direct coupled-voltage amplifier whose function it is to amplify the voltage applied to the grid of V_3 as much as possible. Triodes of the 6F5 type can be used for this purpose, but their voltage amplification is usually quite small compared to that obtainable with the 57-6J7 family of pentodes. The 57 tubes have been found to work quite satisfactorily with E_i voltages of 3700 volts, using 0.25 megohm and higher values for R_5. (As many as eight 2A3 tubes in parallel have been used for V_4 with 1 megohm for R_5 and 3700 volts for E_i.) With low-voltage operation it is convenient to use the 6J7 or 6J7G tubes, but with high-voltage operation it is more convenient to use the 57 tube because filament transformers with the proper insulation are more readily available for this tube. The possibility of using the 802 transmitting pentode for this purpose, especially at the higher voltages, should not be overlooked.

As explained before, the stable operating voltage E_0 is determined by the values of the voltage E_s and the resistors R_1 and R_2.

$$E_0 = \frac{(R_1 + R_2)}{R_1} E_s.$$

If it is desirable to have a variable voltage E_0, R_1 and R_2 can be broken up into two fixed resistors R_3 and R_4 and a potentiometer R_p. If the maximum and minimum values of E_0 are chosen together with any convenient value of R_p, then the values of R_3 and R_4 may be calculated in the following manner:
Since

$$E_{0\max} = \frac{(R_3 + R_p + R_4)}{R_3} E_s$$

and

$$E_{0\min} = \frac{(R_3 + R_p + R_4)}{R_p + R_3} E_s$$

therefore,

$$R_3 = \frac{R_p}{\dfrac{E_{0\max}}{E_{0\min}} - 1}$$

and

$$R_4 = R_3\left(\frac{E_{0\max}}{E_s} - 1\right) - R_p.$$

The value of E_s required in the equations above may be estimated very closely by subtracting from E_R slightly less than the cutoff voltage of V_3. As an example, if a VR105-30 tube is used for V_2, E_R is equal approximately to 107 volts with 25 milliamperes flowing through it. If a 6J7 tube is used for V_3 with a screen voltage of about 100 volts, 6 volts on the grid just fails to produce cutoff in V_3. From these values it can be estimated that E_s should be about 101 volts. The correspondence between design data using this value of E_s and the actual voltage ranges obtained, as will be explained later, shows that this is a very good approximation to the correct value.

If it is desired to have several ranges over which the voltage may be varied with the one potentiometer, then calculations of R_3 and R_4 can be made as indicated before for each individual range, and the composite values of R_3 and R_4 may be obtained by means of a multigang selector switch.

The exact amount of change of E_0 that is applied to the grid of V_3 depends upon the ratio $R_1/(R_1+R_2)$, and if this ratio is very small, unreasonably poor regulation will result. One good way of increasing this ratio for any particular value of E_0 is to increase the value of E_R either by using a different type of tube or else by using two or more of the voltage-regulator tubes in series for V_2. One of the disadvantages of increasing E_R is that it raises the value of the minimum E_0 that it is possible to obtain. This limitation will be explained later on. The sacrifice in the minimum value of E_0 obtainable is usually well compensated for by the improved regulation that results.

Practically all of the variation in E_0 due to ripple voltage can be applied to the grid of V_3 by shunting R_2 with the condenser C_4. The capacitive reactance of this condenser should naturally be small compared to the resistance R_1 at the ripple frequency. If condenser C_4 is used, the regulator circuit will become considerably more sensitive to periodic variations of E_0 at ripple and higher frequencies than to other variations.

While there are several types of tubes that can be used for V_4, the most satisfactory one appears to be the 2A3 due to its large current-handling capacity and exceedingly low plate resistance. The operation of this tube in voltage-regulator circuits usually carries it into the region beyond that required for ordinary amplifier operation and for this reason a set of plate characteristics for one of these tubes, up to 950 volts on the plate, is shown in Fig. 2. Since the permissible plate dissipation on the 2A3 tubes is not ordinarily specified by the tube manufacturers, a test was run on one of the modern types of this tube. It was found that the plates began to show a visible glow in a dark room when the plate dissipation attained a value between 35 and 40 watts. It is concluded from this test that it

should be permissible to design the circuit, for voltage-regulator operation, using maximum plate dissipations on the 2A3 tubes of about 25 watts.

It is very often found that the full desirable range of the voltage regulator cannot be attained without exceeding the maximum permissible plate dissipation of the V_4 tube. This problem can be very easily overcome by connecting two or more of these tubes, as may be required, in parallel. In order to avoid parasitic oscillations that may develop between the parallel tubes, it may be desirable to connect a small amount of resistance in series with the individual plates or grids of the tubes.

If the use of several tubes in parallel for V_4 is still not sufficient to cover the desired voltage variation, or if a closer control of the voltage is desired, then it may be convenient to split the over-all voltage range into minor ranges and to change the transformer input voltage suitably when the ranges are changed.

For any combination of this regulator circuit there is a definite minimum output voltage which can be obtained and which depends to a large extent upon the values of E_R, E_i, and the plate current for the individual V_4 tubes. The manner of estimating this voltage can best be shown by the following example: assume E_R to be 100 volts and E_i to be of such a magnitude that the plate voltage of the tube V_4 is 600 volts, and find the minimum value of E_0 for a current of about 1 milliampere through the individual 2A3 tubes used for V_4. Reference to Fig. 2 shows that the voltage of the grid of V_4 must be 160 volts negative with respect to its cathode. Assuming as an extreme case that the plate voltage of V_3 is only 20 volts, this means that the cathode of V_4 must be $100+20+160=280$ volts above the negative side of the line and, therefore, any further adjustment of the R_1-R_2 combination tending to reduce E_0 will only drive the grid of V_3 positive without reducing E_0. In choosing the minimum value of E_0 required on the lowest range, therefore, care should be taken that this voltage is not less than the minimum voltage permitted by the circuit. As mentioned before, it is usually desirable to sacrifice some of the minimum value of E_0 obtainable in favor of improved regulation by increasing the value of E_R.

If the minimum current involved in the above problem were greater than 1 milliampere, there would be a corresponding decrease in E_i, the plate voltage of the V_4 tube, and the grid voltage required on this tube. All of these changes would tend to reduce, slightly, the minimum voltage obtainable. For this reason it would be desirable, in practice, to bleed a small amount of current on the output side of the regulator circuit in addition to that bled by the R_1-R_2 combination. A voltmeter across the output terminals would help from this point of view and also would indicate the correct output voltage at all times.

A discussion of the rectifier circuit would not normally be in the province of this paper, but since the ability of the regulator to maintain a constant output voltage depends on the burden placed on it, a short review of the factors that improve rectifier regulation will be given here.

It is a well-known fact that the regulation of a rectifier with a choke input filter is superior to that obtained when a condenser input filter is used. In order to obtain these benefits, however, it is essential that

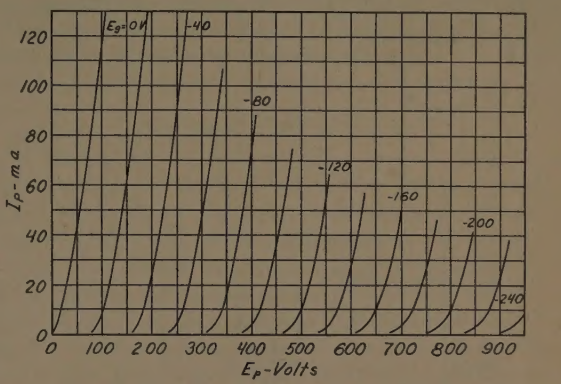


Fig. 2

operation of the circuit be limited to the "flat" portion of the output characteristic. Operation along this portion of the curve can be insured by a correct proportioning of the first choke and the over-all bleeder current (minimum current drawn from the rectifier) so that the inductance of the choke $L=E_i/I_{\text{bleeder}}\times 1/1131$ henries. It might be well to point out here that this is the inductance required at the actual operating conditions and not the inductance at zero direct current as rated by some of the manufacturers. As mentioned before, the bleeder current is made up of the currents through R_B, the R_1-R_2 combination, the $R_7-R_8-V_2$ combination, and any other permanent load that may be connected across the line.

In the case of high-voltage operation the relative values of L and C_1 may be such that the ripple voltage appearing at E_i is several hundred volts and this naturally would place a large burden on the regulator circuit. In these cases it is usually convenient to use a two-stage choke-input filter with a swinging choke in the first stage and a smoothing choke in the second stage.

The selection of rectifier tubes, of course, depends on the various currents and voltages desired. In the low-voltage range high-vacuum tubes find high favor while in the high-voltage range tubes of the 866A type are indicated. Whenever gaseous tubes are used it is desirable to use a condenser C_3 of about 0.01 to 0.05 microfarad to eliminate the majority of the "hash" originating when the gas tubes "break down."

It is highly desirable, especially in high-voltage applications, that the plate transformer used have as good a regulation as possible and that the filter chokes have a low direct-current resistance.

The voltage required on the secondary of the plate transformer may be readily estimated in the following manner:

direct output voltage of rectifier tubes = maximum output voltage required
+plate voltage of V_4 at maximum current per tube (assuming $E_g = -20$ volts as a safety factor)
+direct voltage drop in filter at maximum current value.

Design and Construction Comparisons

Low-, medium-, and high-voltage power supplies have been designed by the author on the basis of the analysis presented in this paper and a very satisfactory correlation has been found between the design data and the actual performance in all cases. Fig. 3 is the circuit diagram of a medium-voltage-regulated power supply designed to deliver up to 200 milliamperes in a voltage range from 250 to 1350 volts.

The complete range was designed to be covered in four uniform overlapping ranges as shown in Table I (values taken at no load):

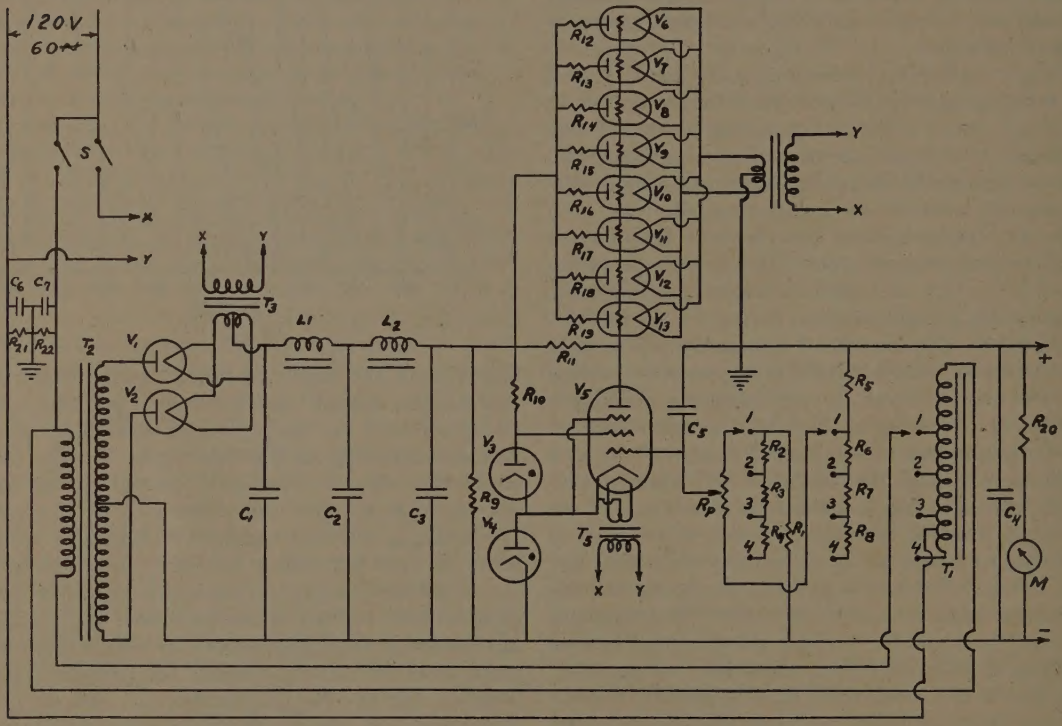


Fig. 3

If gaseous rectifiers are used, the voltage drop across these tubes can be neglected and, correcting for form factor, the full-load secondary voltage of the plate transformer to center tap should be 1.11 × (direct output voltage of rectifier tubes). In order to compensate for the regulation of the transformer, the no-load value of the secondary voltage to the center tap should be taken as being about 5 per cent higher than the full-load value unless more definite information is available.

If several ranges of voltage are required, a tapped autotransformer may be used to provide the correct transformer secondary voltage for each range and the taps of the autotransformer can be changed on the multigang selector switch used to change voltage ranges. In order to catch unavoidable circuit changes that may occur, it is usually desirable to check the design values of the voltage taps with the actual values required, before winding the autotransformer.

TABLE I

Range	Design		Actual	
	$E_{b\min}$	$E_{b\max}$	$E_{b\min}$	$E_{b\max}$
1	250	600	295	603
2	500	850	503	835
3	750	1100	750	1080
4	1000	1350	991	1320

It can be seen from Fig. 3 that the range switching is obtained by means of a three-gang—four-position selector switch and that a separate input voltage is supplied to the plate transformer from a tapped autotransformer on each range. It was essential to change the input voltage on each range in order to avoid excessively large plate dissipations on the 2A3 tubes when the power supply was used on the lower ranges. The design and actual values of voltage of the autotransformer taps are given in Table II. The design

values for the voltage of the autotransformer taps were perfectly satisfactory in all cases, but the values actually used were increased slightly to compensate for line-voltage fluctuations. This increase was permissible in all cases because the number of 2A3 tubes used was conservatively chosen for a maximum plate dissipation of only 18 watts on each tube, and the slight additional plate dissipation could cause no trouble.

TABLE II
VOLTAGE OF AUTOTRANSFORMER TAPS

Range	Preliminary Design	Actual Values Used
1	65	68
2	85	88
3	105	108
4	125	128

The actual proof of the effectiveness of the regulated power supply depends, of course, on how constant the output voltage can be kept with variations in load current. The values for the maximum and minimum settings of the third range, shown in Table III, are typical of all the ranges and were considered to be satisfactory for all practical purposes. The output ripple voltage obtained in the above tests was in all cases negligibly small.

TABLE III

Maximum Setting of Range 3						
I_{dc} (ma)	7	18	28	82	132	185
E_o (volts)	1080	1080	1080	1080	1079	1078
Minimum Setting of Range 3						
I_{dc} (ma)	5	13	22	59	131	202
E_o (volts)	750—	750—	750—	749	748	748

Eight 2A3 tubes were used in parallel for control purposes and, as mentioned before, the maximum plate dissipation per tube was conservatively designed to be only 18 watts. The resistors $R_{12}-R_{19}$, connected in the plate circuits of the individual 2A3 tubes, are each a 100-ohm, 1-watt carbon resistor and are for the purpose of eliminating any tendency towards parasitic oscillations that may exist in the circuit.

The condenser C_1, connected from the center tap of the V_1-V_2 filament transformer to the center tap of the plate transformer, is a 0.015-microfarad condenser and is intended to reduce the "hash" that is present when rectifier tubes of the 866A type are used. The effect of this condenser is negligibly small as far as the "choke input" filter is concerned.

In all high-voltage applications, and especially where gaseous rectifiers are used, it is desirable to bring the filaments up to normal operating temperatures before the plate voltage is applied. It is also desirable to turn off the plate voltage before the filament power is removed. This point is taken care of by the stepped switch S which closes the filament circuit first and the plate circuit next, reversing the process when the switch is opened. A time-delay relay could be used with equal success for this purpose.

It can be seen from the circuit diagram that the positive side of the line has been grounded. This is the normal operating condition for many of the high-voltage applications, but the negative side of the line could have been grounded instead if the operation had required it.

In order to allow static charges that may build up between the alternating-current line and the regulated power supply to leak off, and to minimize slight line disturbances, it is desirable to have a resistance-capacitance combination as indicated by C_6, C_7, R_{21}, and R_{22}.

Constructional Considerations

In the case of low-voltage—low-current regulated-power supplies, it is quite convenient to place the whole rectifier and regulator on one chassis. When higher voltage and current combinations are required, the use of a single chassis becomes inconvenient and the use of a rack with two decks offers definite advantages. In this case the rectifier circuit and most of the heavy filtering equipment can be placed on the lower deck, while the comparatively lightweight regulator equipment can be placed on the upper deck. Connections between the two decks can be obtained by means of a multiple-conductor cable and a plug and socket.

Conclusions

In summarizing, a sequence of design steps may be pointed out although the order of the sequence need not be followed rigorously. Referring to Fig. 1:

1. Choose maximum and minimum output voltage required and subdivide into several ranges, if desired, with a generous amount of overlap.

2. Considering the voltage ranges chosen in step 1, decide upon the type and number of control tubes V_4 required in parallel within their permissible plate dissipation. If the number of tubes required is unreasonably large, a revision of the choice in step 1 may be necessary.

3. Upon the choice of suitable tubes V_2 and V_3 and potentiometer R_p, calculations can be made to determine the required values of R_3 and R_4 for each of the ranges chosen in 1.

4. Choose the values of the resistors R_5, R_6, and R_7 with special attention to the possibility of using a voltage-regulator tube in place of R_6 if E_i is varied appreciably for the various ranges.

5. Choose appropriate values of L and R_B to insure operation on the "flat" region of the rectifier output characteristic for all ranges.

6. Choose appropriate values of C_1, C_2, and C_4 depending on the maximum voltages expected, and if the ripple-voltage input to the regulator circuit is too high, an additional filter stage should be added.

7. Upon choosing appropriate tubes for V_1 calculate the secondary voltage required on the plate transformer for the particular range involved and, if

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necessary, supply the primary of the plate transformer from a tapped autotransformer.

8. If gaseous rectifiers are used, choose a suitable value for C_3.

9. Add any refinements to the circuit that the particular application may demand.

10. Provide appropriate physical arrangement of parts on a chassis being careful to furnish adequate insulation where required.

The above procedure may be revised appropriately if a specific plate transformer is available and it is desired to obtain optimum performance from it.

The Measurement of Transcription-Turntable Speed Variation*

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Summary—*Speed constancy or freedom from speed fluctuation ("wows") is becoming more important due to the widespread use of records in radio broadcasting. Equipment of a simplified nature which will evaluate the wow content as a single figure is needed for standardization purposes. Some of the existing equipment is reviewed, and the importance of having a meter with proper ballistic constants for measuring the wow content is shown.*

BECAUSE of the increase in the use of transcription and home-phonograph records in the radio broadcasting field, it is becoming more important to maintain high standards of disk reproduction; one of the requirements of faithful reproduction is that of maintaining a constant speed of the reproducing turntable. It is not only necessary that the average speed as indicated by an ordinary speed-measuring device be the same as that of the recording turntable in order to maintain the correct pitch, but it is important that any instantaneous deviations from this average speed be kept at a minimum, for it is these deviations, heard as "wows," that are particularly objectionable.

As perceived by ear, the effect of these deviations from average speed depends upon the percentage change and the rate of variation. Published data[1] have shown that pitch variations as low as 0.3 per cent are detectable by ear in a 1000-cycle tone when the variations occur at rates of 1 to 3 cycles per second. If the cycle of pitch variation is either faster or slower, the ear becomes less sensitive to pitch changes. The above tests were conducted with earphones and so independent of the acoustic characteristics of the room. Although a dead room has little effect, a live room increases the apparent sensitivity to pitch changes, because of an amplitude pulsation created by shifting of a standing-wave pattern.[2,3] This amplitude pulsation, however, is not considered too objectionable as it is often purposely introduced by the musician, and known as tremolo, and so it is believed justifiable to use the earphone data in considering pitch variations only. At low rates of variation the change is perceived directly as a pitch variation, and at rates such as once around of the turntable at 78 revolutions per minute, the variation resembles so much the spoken sounds "wow-wow," that the word "wow" is generally used to denote such speed variations. As the rate of variation is increased it is heard as a flutter and, finally, all sense of pitch variation is lost; but if a constant-note record is reproduced, sidebands not harmonically related will be heard. The presence of this distortion can be explained theoretically as resulting from frequency modulation produced by the variation in speed.[1,2,4]

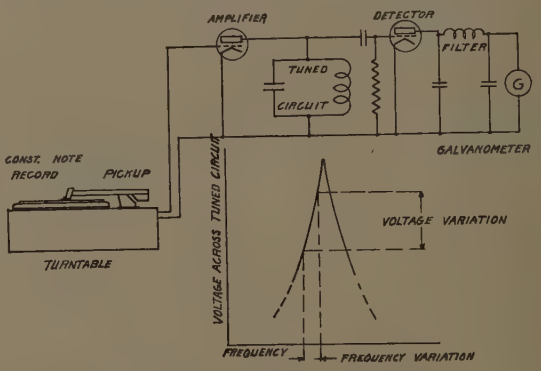

Fig. 1—Method of making wow measurements.

Realizing the importance of wow-free turntables for record reproduction, the National Association of Broadcasters in its standardization program is undertaking the task of setting wow limits on disk recording and reproducing equipment. Since equipment and methods of measurement must be established before limits can be set and maintained, the writer, as a member of the standards subcommittee, has undertaken a

* Decimal classification: 621.385.97. Original manuscript received by the Institute, June 26, 1942. Presented, Summer Convention, Cleveland, Ohio, June 29, 1942.

† RCA Manufacturing Company, Indianapolis, Indiana.

[1] E. G. Shower and R. Biddulph, "Differential pitch sensitivity of the ear," *Jour. Acous. Soc. Amer.*, vol. 3, pp. 275–287; October, 1931.

[2] T. E. Shea, W. A. MacNair, and V. Subrizi, "Flutter in sound records," *Jour. Soc. Mot. Pic. Eng.*, vol. 25, pp. 403–415; November, 1935.

[3] W. J. Albersheim and D. MacKenzie, "Analysis of sound film drive," *Jour. Soc. Mot. Pic. Eng.*, vol. 37, pp. 452–454; November, 1941.

[4] E. W. Kellogg and H. Belar, "Analysis of the distortion resulting from sprocket hole modulation," *Jour. Soc. Mot. Pic. Eng.*, vol. 25, pp. 492–502; December, 1935.

review of some of the existing wow-metering equipment. One of the first wowmeters was designed about 1929 by M. S. Mead of the General Electric Company for testing sound film reproducers. A few years later, the writer developed equipment based on the same principle of operation for measuring turntable-speed variations. For these measurements (Fig. 1) a 1000-cycle signal reproduced from a constant-note record on a turntable was amplified and applied across a circuit tuned to a frequency slightly higher than the record frequency so that operation was on one side of the resonance curve. Thus, whenever the turntable speeded up and increased the frequency from the constant-note record, the voltage across the tuned circuit increased,

Fig. 2—Magnetic tone wheel and pickups.

and whenever the turntable slowed down and the frequency decreased, the voltage across the tuned circuit decreased. In this way the frequency-modulated signal due to variations in turntable speed was converted into an amplitude-modulated signal. This amplitude-modulated signal was then rectified, the 1000-cycle carrier filtered out, and the modulation corresponding to the variations in turntable speed measured by observing the deflection of a galvanometer light beam.

The equipment[5] now in use in RCA laboratories for studying the action of turntables is basically the same but has been improved in many respects. Two tuned circuits are now used, one tuned above the operating frequency and the other below. This provides a push-pull arrangement which smooths out and extends the operating range, besides making the equipment relatively insensitive to the voltage changes that accompany the frequency variations when a velocity-responsive pickup is used.

[5] E. W. Kellogg and A. R. Morgan, "Measurement of speed fluctuation in sound recording and reproducing equipment," *Jour. Acous. Soc. Amer.*, vol. 7, pp. 271–280; April, 1936.

In place of records which wear out and vary in wow content among pressings, making it hard to duplicate results, a magnetic tone wheel is used. This tone wheel (Fig. 2) consists of a number of laminations with machined teeth around the periphery, clamped to-

Fig. 3—Amplifier and equipment.

gether in the rim of a cast aluminum wheel. Two pickups are mounted diametrically opposite to each other on a bridging structure which is supported from a spindle centrally located on the tone wheel. These pickups have pole pieces with corresponding teeth and are separated from the tone-wheel laminations by a small air gap. The output coils are connected in series and the pole pieces are carefully adjusted for equality of phase and amplitude. As a result of the carefully planned and executed design, errors due to construction and misalignment during setup have been reduced to a minimum and measurements are readily duplicated. The tone wheel is used at both 78 and 33.3 revolutions per minute, the resulting frequency at the latter speed being 426 cycles per second and the former 1000 cycles. The tuned circuits may be adjusted to operate at either frequency. As before, measurements are made by observing the deflection of a

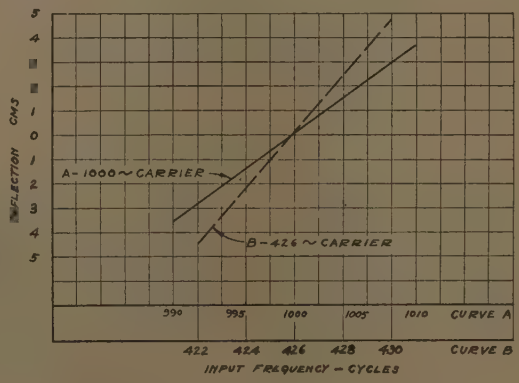

Fig. 4—Static frequency calibration.

galvanometer light beam and oscillograms or wowgrams of the speed variations are usually made.

A picture of the amplifier and associated equipment mounted in a portable cabinet is shown in Fig. 3. A film drum used for taking the wowgrams may be seen on top of the cabinet and the motor used to drive it is mounted on the side of the amplifier case.

The curves for static calibration of the equipment at 426 and 1000 cycles per second are shown in Fig. 4.

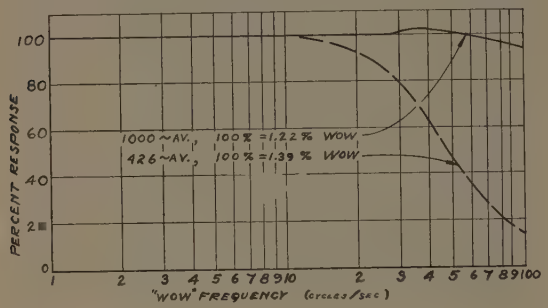

Fig. 5—Response to rate of wow variation.

Note that in both cases the calibration is linear for a range of about ±1 per cent. This range has been found adequate for most measurements.

Fig. 5 shows the response of the equipment to rate of variation of the wow. The loss of response at the higher frequencies for the 426-cycle carrier is due to the low-frequency cutoff of the filter necessary to eliminate the carrier.

Fig. 6 shows some typical wowgrams. The record covers a period of approximately 5 seconds. Note that the wave shape is irregular and complex. It is possible to pick out the frequency of the wow components and measure their amplitudes and, knowing the frequency of the rotating parts of the driving system, determine which part is causing excessive wow.

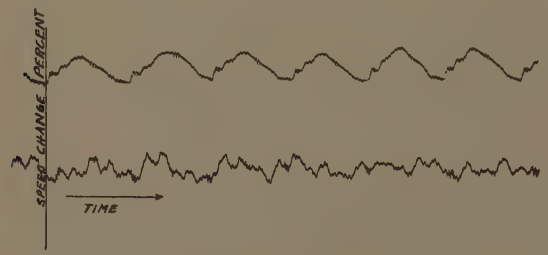

Fig. 6—Typical wowgrams.

The equipment which has been described was made up primarily for laboratory and diagnostic purposes and not to evaluate the wow content in terms of a single figure of merit. Where an over-all measurement is wanted, equipment of a simplified and less expensive nature would be more desirable.

There are a number of wowmeters or flutter bridges in use which give an approximate root-mean-square value of the deviations in speed, and it is believed that this type of instrument is more suitable for the broadcaster's application. An instrument of this type has been described by Scoville.[6]

Fig. 7 shows the circuit of a simple instrument of the bridge-circuit type used extensively by the RCA service department. In operation the capacitor-inductor branch is tuned to the average carrier frequency and any departure therefrom unbalances the bridge and so registers on the meter. A band-pass filter in the input circuit eliminates harmonics and low-frequency disturbances such as hum, which would otherwise affect the reading of the meter.

With the bridge balanced, a deviation in frequency either way causes an increase in voltage across the output terminals and so imparts a double-frequency swing to the meter. The ballistic constants of the meter are important where low wow rates correspond-

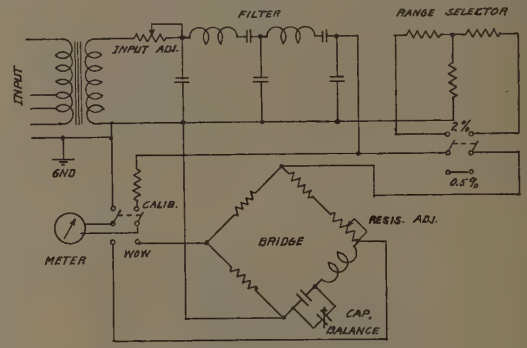

Fig. 7—Bridge-circuit type of wowmeter.

ing to once around at 33⅓ and 78 revolutions per minute, or approximately ½ and 1.3 cycles per second, are involved.

If the meter is so designed that the deflection of the needle is proportional to the square of the current, it might be legitimate to take the estimated average deflection as an indication of the magnitude of the root-mean-square flutter or wow, but in practice such an estimate is difficult and unreliable unless the swings are quite small compared with the average deflection. If the meter needle resonates at the wow frequency, the swings may become so large that it is out of the question even to estimate a fair average deflection.

Tests were made, and the results given in Fig. 8 show the maximum and minimum swing of the pointer with varying wow rates for a vu meter, used for measuring volume levels, a rectifier type of microammeter and a thermocouple meter. It would be difficult to read either of the rectifier meters accurately at these low wow rates due to excessive swinging of the pointer and the reading would be an indication of the meter characteristic rather than the wow.

A rectifier meter with a lower mechanical resonance and increased damping was obtained and the results

[6] R. R. Scoville, "A portable flutter-measuring instrument," *Jour. Soc. Mot. Pic. Eng.*, vol. 25, pp. 416–422; November, 1935.

are shown in Fig. 9, along with those obtained with the vu meter. The swing of the pointer of this meter is small enough to permit a fairly accurate estimate of an average reading, which is representative of the wow rather than of the meter characteristics. Either a tone wheel or a record can be used for generating the carrier signal. A record would include the effects of vibration of the pickup, since any to-and-fro sliding motion of the stylus in the groove due to vibration of the tone arm produces a wow the same as does irregular turntable rotation. A tone wheel, such as the one described, with balanced pickups supported by a spindle on the tone wheel, is relatively unaffected by vibration, does not have to be carefully centered on the turntable, and is especially suitable for measuring pure speed variations. A record gives a more representative over-all measurement of general turntable performance, but suffers the disadvantage of wearing out and introducing errors due to recording and processing.

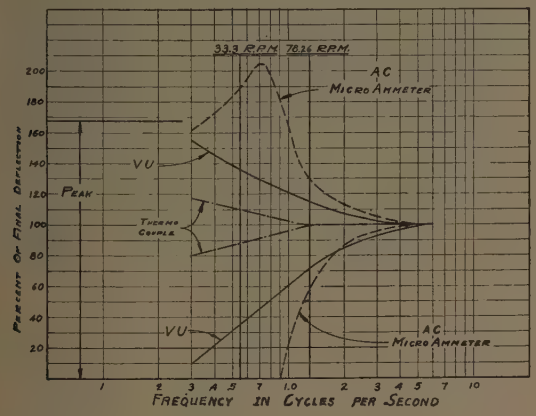

Fig. 8—Meter response.

The fact that wowmeters in use in RCA laboratories gave results in the form of oscillograms resulted in the practice within the company, and later outside, of expressing the wow content as a single figure in terms of peak-to-peak values. That is, the difference between the maximum and minimum speeds attained during the period covered by the oscillogram (6 seconds) is expressed as a percentage of the average speed. This is a convenient, but perhaps not an ideal, means of expressing the wow content, since it gives equal rating both to a speed variation which only occasionally reaches the peak value and to a periodic variation of the same peak value. The wowgrams in Fig. 6 illustrate this point. The peak-to-peak value in both cases is the

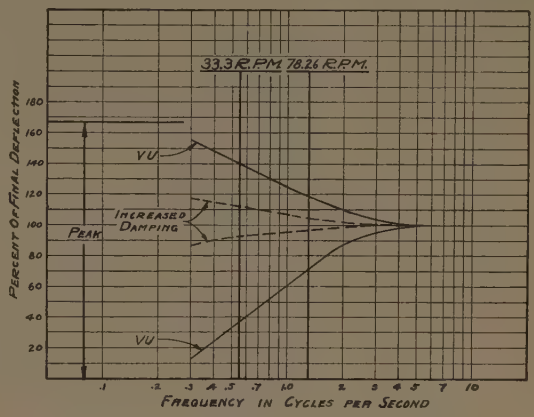

Fig. 9—Meter response with well-damped meter.

same, but the average is quite different. It seems reasonable that the wow obnoxiousness depends upon the duration of the deviation from the average speed, within limits of course, and upon the amplitude or per cent of deviation. It is believed that the obnoxiousness is not a linear function of the amplitude or percentage deviation, but increases at some rate greater than the first power. If this is true, then a root-mean-square reading would probably be a good way of expressing the wow content as a single figure. For a true root-mean-square reading, a thermocouple meter should be used, since wowgrams show the speed variation to be complex and far from a simple sine variation. A thermocouple, however, is delicate and does not readily withstand overload, so a more rugged type of meter is desirable. An approximate root-mean-square value as obtained with a rectox meter would probably prove satisfactory for all practical purposes. The ballistic characteristics are important and should be standardized to insure comparable readings.

Loop Antennas for Aircraft*

GEORGE F. LEVY†, ASSOCIATE, I.R.E.

Summary—*While the theory of operation and the procedure for measurement are similar for all loop antennas, the electrical and mechanical design of aircraft loop antennas differs considerably from that of the other types. In this paper those characteristics, requirements, and design considerations which are associated uniquely with aircraft loop antennas operating in radio range or beacon band extending from 200 to 400 kilocycles will be discussed.*

In the case of aircraft loop antennas it is necessary to satisfy three important requirements. The first requirement is that a loop antenna form a highly efficient portable antenna; the second, and most important requirement, is that the loop be capable of being used as an accurate direction finder; and third, an electrostatically shielded loop antenna is an effective way of decreasing precipitation or snow static which is quite bothersome to radio reception on aircraft.[1,2]

The "low-impedance" and the "high-impedance" types of air-core aircraft loops are considered in detail. Both types are analyzed mathematically on the basis of their receiving efficiency and directive properties. Actual polar-characteristic curves are given for a number of loop antennas of both types. Iron-core loop antennas which have been used quite extensively abroad are considered separately and comparison is made with the more widely used air-core types.

Introduction

IN ORDER to receive signals the loop must, of course, be placed outside of the fuselage in the case of all-metal airplanes, and it is desirable to place it outside the metal framework in the case of fabric-covered airplanes. Mounting it away from the fuselage

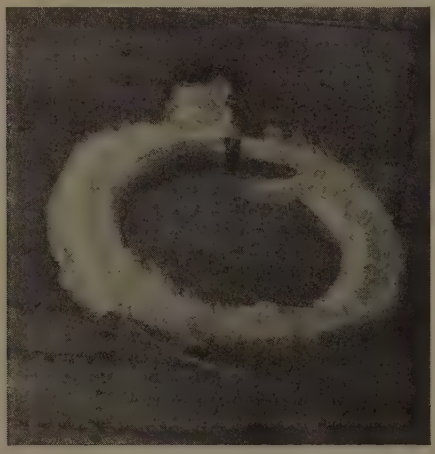

Fig. 1—Icing of a loop antenna.

puts the loop antenna into the air stream where it presents an appreciable aerodynamic drag on the airplane and is itself subjected to a considerable distortive force. When commercial aircraft speeds of over 200 miles per hour are considered, this drag represents a substantial loss of horsepower. Icing of the loop antenna under certain conditions further complicates the problem. Fig. 1 clearly shows a moderate accumulation of ice on a typical aircraft loop antenna. Thus it has been found that drag and icing considerations limit the size of rotatable loops having self-contained shielding to a maximum diameter of about 12 inches.

Another method is employed to reduce aerodynamic drag and icing and at the same time relieve the loop antenna itself of strain, and that is to use a streamlined housing over the loop. Example of such construction is illustrated in Fig. 2. The electrostatic shield may be

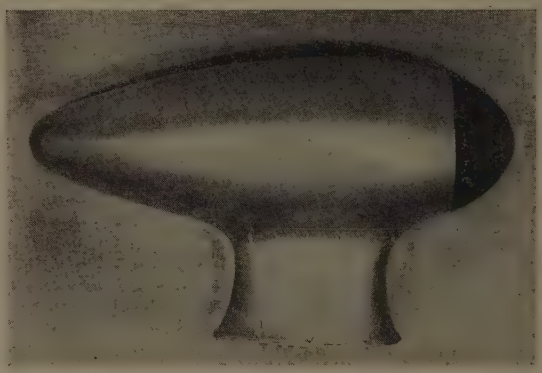

Fig. 2—Streamlined loop-antenna housing.

made a part of the loop or may be incorporated in the housing. Removing the shield from close proximity to the loop winding will result in a substantial improvement in the Q of the loop antenna.

In addition to aerodynamic drag and icing, the problem of moisture absorption is also present. This problem is very important as it directly affects the electrical performance of the loop antenna by lowering the Q of the loop and its connecting transmission line. The moisture problem is greatly aggravated by the changes in altitude and hence, changes in pressure to which the loop is subjected. To maintain the Q at its maximum value under all kinds of flying conditions requires the hermetical sealing of the loop antenna and the connecting transmission lines.

The remaining mechanical problem to be considered in the design of loop antennas is that of rotating and indicating mechanisms. The loop gear box, besides satisfying certain mechanical requirements, must also possess certain electrical characteristics. It is very essential that the parallel capacitance presented by the gear box be as small and low-loss as possible.

Aircraft loop antennas can be definitely divided into two types which have come to be commonly called, in aircraft loop parlance, the "low-impedance" loop and the "high-impedance" loop. The main distinguishing

* Decimal classification: R525. Original manuscript received by the Institute, March 30, 1942; revised manuscript received, November 23, 1942. Presented, Winter Convention, New York, N. Y., January 12, 1942.

† Communications Engineer, United Airlines, Chicago, Illinois.

[1] H. K. Morgan, "Rain static," PROC. I.R.E., vol. 24, pp. 959–963; July, 1936.

[2] H. M. Hucke, "Precipitation static interference on aircraft and at ground stations," PROC. I.R.E., vol. 27, pp. 301–316; May, 1939.

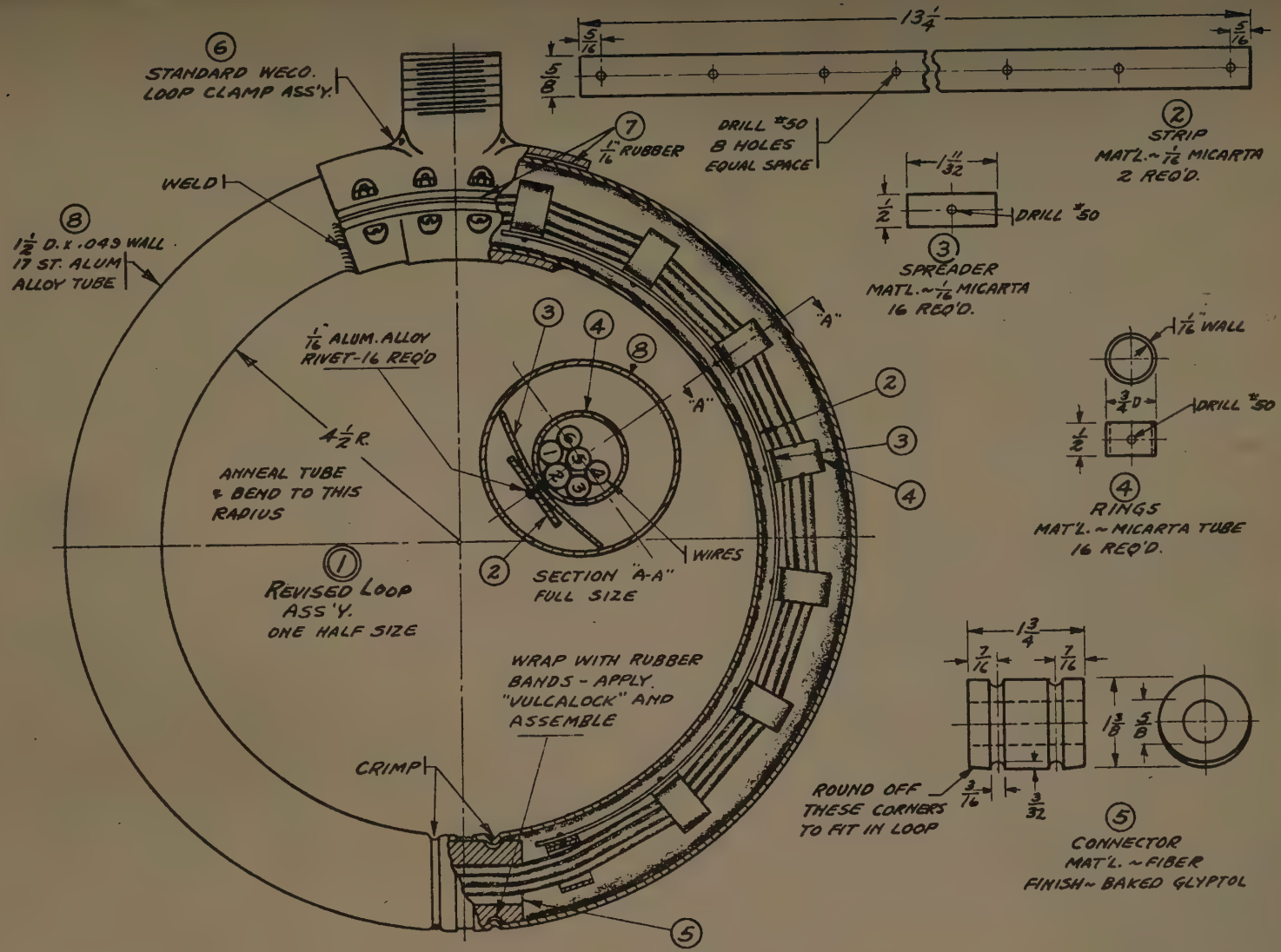

Fig. 3—Constructional details of a low-impedance-loop.

feature is the number of turns employed in the winding. The low-impedance-loop winding generally runs anywhere from four to twenty turns, whereas the high-impedance-loop winding may run from twenty to as high as seventy turns. The constructional details of a typical low-impedance loop are illustrated in Fig. 3. Because of the difficulties of hermetically sealing high-impedance loops and of constructing and installing low-loss and low-capacitance transmission lines, high-impedance-loop installations are rarely used on transport and military aircraft.

Signal Pickup Characteristics of Aircraft Loop Antennas

Consider now the electrical aspects of the high- and low-impedance-loop systems. In the high-impedance case the loop has sufficient inductance so that it is always tuned to the incoming frequency by a condenser connected directly across it. By simple series-circuit analysis, as shown in Fig. 4, it can be seen that the

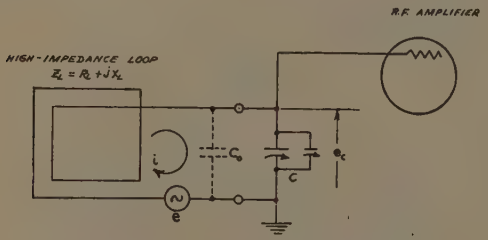

Fig. 4—High-impedance-loop input circuit.

h = effective height of loop in meters
ϵ = field strength in millivolts per meter
C_0 = distributed capacitance of loop

$$e = h\epsilon$$

(at resonance) $$i = \frac{e}{R_L}$$

$$e_C = \frac{e}{R_L} X_C = \frac{e}{R_L} X_L$$

but $$Q_L = \frac{X_L}{R_L}$$

$$\therefore \quad e_C = Q_L e$$

voltage appearing across the condenser C is equal to Qe or directly proportional to the Q of the loop. It is desirable to obtain a very high value of loop Q for two reasons. A high input voltage will be obtained to the radio-frequency amplifier, and by virtue of the higher radio-frequency grid voltage, the signal-to-noise ratio of the receiver will be increased. In order fully to realize the advantages of a high-impedance loop, the loop is fed directly into a radio-frequency amplifier, which is located very close to the loop. The output of the amplifier may then be connected to the remainder of the receiver through a low-impedance, low-loss transmission line. If it is desired to couple the loop to the receiver with a high-impedance line, the line necessarily must be short and fixed in length for all installations. The losses in the line and its capacity will decrease the Q of the loop, and reduce the available tuning range, respectively. Alternatively, if an attempt is made to match the tuned loop circuit to a low-impedance transmission line, it will be found that the voltage step-up obtained by virtue of the high Q of the loop will have disappeared.

If it is desired to couple the loop to the receiver via a low-impedance line without an intervening amplifier, then the best arrangement is to use a low-impedance loop connected directly to a low-impedance concentric transmission line with a matching transformer at the receiver input. Fig. 5(a) shows the actual physical

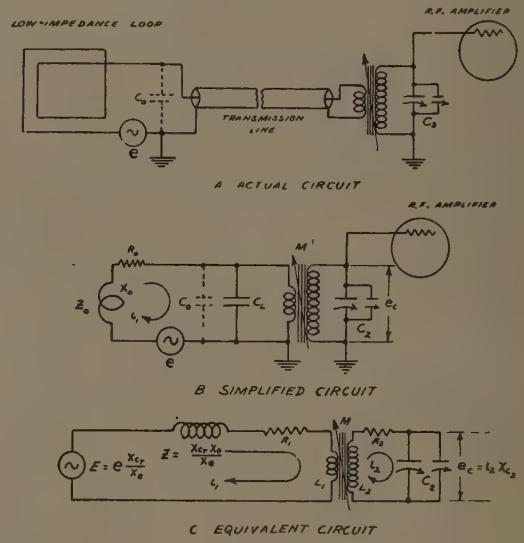

Fig. 5—Low-impedance-loop input circuit
(a)—Actual circuit of a low-impedance-loop system.
(b)—Simplified circuit of a low-impedance-loop system.
(c)—Equivalent circuit of a low-impedance-loop system.

circuit of a low-impedance-loop system. Mathematical analysis of this circuit is considerably more involved than that of the high-impedance case and numerous engineering approximations must be made. This diagram may be simplified to Fig. 5(b) by considering the transmission line as equal to a lumped capacitance C_L. The series inductance and resistance are very small and may be neglected. This approximation is permissible because the line is very short as compared to the wavelength.

By Thevenin's theorem, the voltage impressed across the primary of the transformer of the simplified circuit is

$$E = \frac{-j\dfrac{e}{\omega C_T}}{R_0 + j\omega L_0 - \dfrac{j}{\omega C_T}} \quad \text{where } C_T = C_0 + C_L. \qquad (1)$$

Rationalizing and combining terms, (1) becomes

$$E = e\frac{-\frac{1}{\omega C_T}\left(\omega L_0 - \frac{1}{\omega C_T}\right) - \frac{jR_0}{\omega C_T}}{R_0^2 + \left(\omega L_0 - \frac{1}{\omega C_T}\right)^2}. \quad (2)$$

By substituting actual circuit constants used at the frequencies being considered, it is found that R_0^2 and $R_0/\omega C_T$ are very small and can be neglected without impairing the engineering accuracy. Making this approximation and setting

$$\omega L_0 - \frac{1}{\omega C_T} = X_0 - X_{C_T} = X_e$$

(2) becomes

$$|E| = e\frac{X_{C_T}}{X_e}. \quad (3)$$

Using the same theorem, the equivalent impedance obtained at the primary terminals looking back toward the loop antenna is

$$Z = \frac{-\frac{j}{\omega C_T}(R_0 + j\omega L_0)}{R_0 + j\left(\omega L_0 - \frac{1}{\omega C_T}\right)}. \quad (4)$$

Rationalizing and combining terms, (4) becomes

$$Z = \frac{-\frac{j}{\omega C_T}(R_0 + j\omega L_0)\left[R_0 - j\left(\omega L_0 - \frac{1}{\omega C_T}\right)\right]}{R_0^2 + \left(\omega L_0 - \frac{1}{\omega C_T}\right)^2}. \quad (5)$$

Using the same procedure as in (2) it is found that terms R_0^2 in the denominator and R_0L_0/C_T and $-jR_0^2/\omega C_T$ in the numerator can be discarded thereby simplifying (5)

$$Z = \frac{-\frac{j}{\omega C_T}(R_0 + j\omega L_0)}{\omega L_0 - \frac{1}{\omega C_T}} \quad (6)$$

$$|Z| = \frac{\frac{1}{\omega C_T}\sqrt{R_0^2 + \omega^2 L_0^2}}{\omega L_0 - \frac{1}{\omega C_T}} = \frac{X_{C_T}X_0}{X_e}. \quad (7)$$

Using the new voltage E and the new equivalent impedance Z, obtained by Thevenin's theorem, the final equivalent circuit may be drawn as shown in Fig. 5(c). Using the notation shown, the circuit equations for Fig. 5(c) may be written

$$E = i_1Z_P - j\omega M i_2 \quad (8)$$

$$0 = -j\omega M i_1 + Z_S i_2 \quad (9)$$

where

$$Z_P = R_P + jX_P = Z + Z_1 = R_1 + j\left(X_{L_1} + \frac{X_{C_T}X_0}{X_e}\right) \quad (10)$$

$$Z_S = R_S + jX_S = Z_2 + X_S = R_2 + j(X_{L_2} - X_{C_2}). \quad (11)$$

Solving for the secondary current i_2,

$$i_2 = \frac{j\omega ME}{Z_PZ_S - \omega^2M^2} = \frac{j\omega ME}{(R_P + jX_P)(R_S + jX_S) - \omega^2M^2}. \quad (12)$$

Expression (12) may be rearranged to

$$i_2 = \frac{j\omega ME}{(R_P + jX_P)\left[R_S + \frac{\omega^2M^2R_P}{R_P^2 + X_P^2} + j\left(X_S - \frac{\omega^2M^2X_P}{R_P^2 + X_P^2}\right)\right]}. \quad (13)$$

To obtain the condition for maximum secondary current i_2, the absolute value of $|i_2|^2$ is differentiated with respect to the tuning reactance X_{C_2}. The maximum value of i_2 is obtained when

$$X_S - \frac{\omega^2M^2X_P}{R_P^2 + X_P^2} = 0. \quad (14)$$

Equation (13) then becomes

$$|i_2| = \frac{\omega ME}{\sqrt{R_P^2 + X_P^2}\left(R_S + \frac{\omega^2M^2R_P}{R_P^2 + X_P^2}\right)}. \quad (15)$$

Using the approximation method previously stated it is found that

$$R_p^2 \ll X_P^2 \text{ and (15) becomes}$$

$$|i_2| = \frac{\omega ME}{X_P\left(R_S + \frac{\omega^2M^2R_P}{X_P^2}\right)}. \quad (16)$$

But $|e_c| = |i_2|X_{c_2}$ and (16) becomes

$$|e_c| = \frac{ME}{C_2X_P\left(R_S + \frac{\omega^2M^2R_P}{X_P^2}\right)}. \quad (17)$$

Equation (17) also may be expressed in terms of the "figure of merit" or Q as

$$|e_c| = \frac{EQ_PQ_S}{\omega^2MC_2(Q_P + Q_S)}. \quad (18)$$

For purposes of calculation, it is desirable to express e_c in terms of the constants of the coupling transformer. Experimental work with such transformers has shown that in actual transformers

$$\frac{X_{C_T}X_0}{X_e} \approx X_{L_1}. \quad (19)$$

Let

$$K^2 = \frac{\omega^2M^2}{X_{L_1}X_{L_2}}.$$

Equation (18) may then be rewritten as

$$|e_c| = \frac{2EK^2X_{C_2}Q_{L_1}Q_{L_2}}{\omega M(4Q_{L_1} + K^2Q_{L_2})}. \quad (20)$$

It can be readily seen that the expression for e_c is not a simple function of the circuit and transformer constants. It should be restated that the entire loop, transmission line, and coupling-transformer system are always tuned to resonance by condenser C_2, which is located usually within the receiver or otherwise ganged to the other tuning condenser in the receiver.

In the practical design of such a system a low-impedance loop is first constructed with as high a Q as possible commensurate with the mechanical and electrical limitations already discussed. Then a tightly coupled transformer is constructed having a high-Q primary of very closely the same inductance as the loop, and a high-Q secondary whose inductance depends on the degree of coupling attainable. In practice, it has been found that the value of resultant secondary Q and the degree of coupling become a compromise. Consequently, the coefficient of coupling is fixed anywhere from 88 to 95 per cent at which point a high value of resultant secondary Q is still obtained. Increasing the coefficient of coupling beyond this value will decrease the resultant secondary Q very rapidly thereby lowering the over-all voltage step-up. It is only by using coupling transformers embodying the closed iron or cup cores that high values of coefficient of coupling and Q may be obtained. The inner iron core is usually made adjustable so that the inductance can be accurately set for correct alignment with the other circuits at the low-frequency end of the band.

Directional Characteristics of Loop Antennas

A comprehensive treatment of this aspect of loop antennas is very lengthy and beyond the scope of this brief paper. In the following discussion only the most important and pertinent considerations will be brought out. While no mention was made of the two types of loop windings in the preceding discussion of high- and low-impedance loops, there are in general two different types of loop windings; namely, (1) the solenoid type in which all the turns have the same area, and (2) the pancake or spiral-wound type in which the areas of successive turns decrease towards the center of the winding. The two types have different directional characteristics and will be considered separately.

The expression for the effective voltage induced in the loop may be derived by either considering the electrostatic component alone or the electromagnetic component alone of the radiated wave as acting on the loop. In any case the same answer is obtained, which is usually given as

$$E = \frac{2\pi\epsilon NA}{\lambda}\cos\phi \quad (21)$$

where E = effective voltage in millivolts
N = number of turns
ϵ = field strength in millivolts per meter
A = area in square meters
ϕ = azimuth angle
λ = wavelength in meters

If the effective voltage E is plotted against the angle ϕ on polar co-ordinate paper, a double-circle curve will be obtained as shown in Fig. 6(a). This is the theoretical polar pattern of a loop antenna.

For the solenoid type, the actual polar characteristic is distorted from the theoretical pattern by three effects. These are (1) the displacement-current effect, (2) the antenna effect, and (3) the shape effect.[3]

Referring to Fig. 6(b) it can be seen that each successive turn is displaced in space from the preceding turn by distance d, which may be equal to the diameter of the wire or greater than the diameter if the loop is space-wound. Thus for all positions of the loop except for $\phi = 0$ the voltages developed in the successive turns will have a slight phase difference existing between them; by virtue of the capacitance existing between turns, a displacement current will flow across the coil eventually passing through the tuning condenser and producing a signal. Since the loop is tuned, the main loop effect produces a current through the winding which is in phase with the voltage, while the displacement current produces a voltage 90 degrees out of

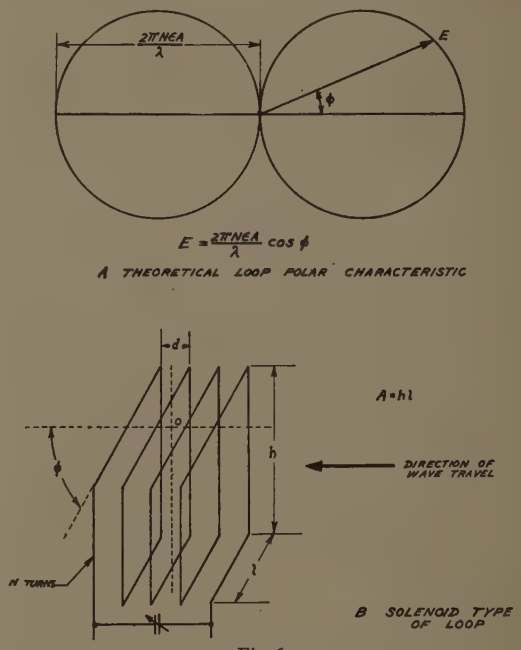

Fig. 6
(a)—Theoretical polar characteristic of a loop antenna.
(b)—Winding diagram of a solenoid type of loop.

[3] A. S. Blatterman, "Theory and practical attainments in the design and use of radio direction findings apparatus using closed coil antennas," *Jour. Frank. Inst.*, vol. 188, pp. 289–362; September, 1919.

phase with the current. In Fig. 7(a) there is shown the distortion of the polar characteristic by the presence of the displacement-current effect.

The antenna effect or vertical component is occasioned by the capacitance of the loop structure to ground causing it to act as a simple vertical antenna. The magnitude of the voltage pickup is a function of the linear dimensions of the loop and is independent of the orientation of the loop, therefore, giving a circular polar pattern. When referred to the axis of the loop, the phase of the antenna-effect voltage does vary with the angular position of the loop. In Fig. 7 (b) the manner in which the polar characteristic is distorted is shown when both the displacement current and antenna effects are present. When the displacement-current effect alone is present, both nulls of the loop

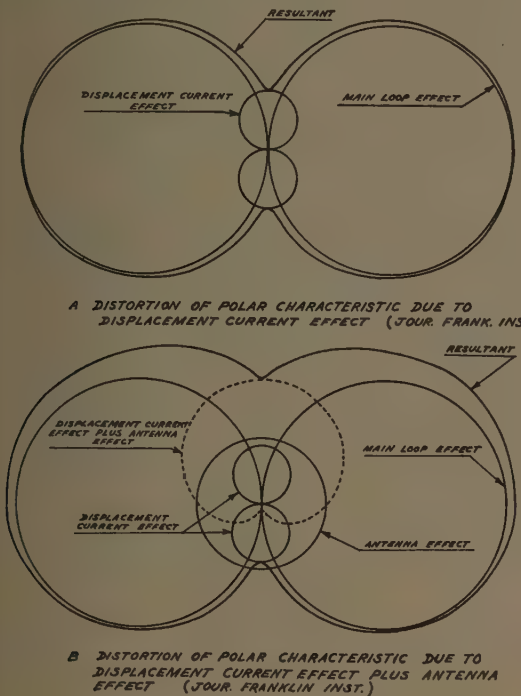
RESULTANT
DISPLACEMENT CURRENT EFFECT
MAIN LOOP EFFECT
A DISTORTION OF POLAR CHARACTERISTIC DUE TO DISPLACEMENT CURRENT EFFECT (JOUR. FRANK. INST)
RESULTANT
DISPLACEMENT CURRENT EFFECT PLUS ANTENNA EFFECT
MAIN LOOP EFFECT
DISPLACEMENT CURRENT EFFECT
ANTENNA EFFECT
B DISTORTION OF POLAR CHARACTERISTIC DUE TO DISPLACEMENT CURRENT EFFECT PLUS ANTENNA EFFECT (JOUR. FRANKLIN INST.)

Fig. 7
(a)—Distortion of polar characteristic due to displacement-current effect.
(b)—Distortion of polar characteristic due to displacement-current effect plus antenna effect.

are broadened by equal amounts while the maxima remain practically unchanged. Introducing both effects simultaneously broadens one null considerably more than the other while the maxima are also increased.

While the two effects just discussed contribute the most distortion to the polar characteristic, there is another effect, while not present in aircraft loops, which further distorts the characteristic of the solenoid type of loop. This effect is known as the shape effect. It has been found that a high narrow winding causes a bulge to appear in the polar characteristic near the vicinity of the minima while a wide low winding causes a flattening of the characteristic at the maxima.

The behavior of the polar-characteristic curve of a loop antenna around the minima is of the most importance. In comparing Figs. 6(a) and 7(a) and 7(b), it can be seen that it is necessary to reduce both the displacement-current effect and the antenna effect in order to secure sharp minima. Both are affected by the shape of the loop. The displacement-current effect for

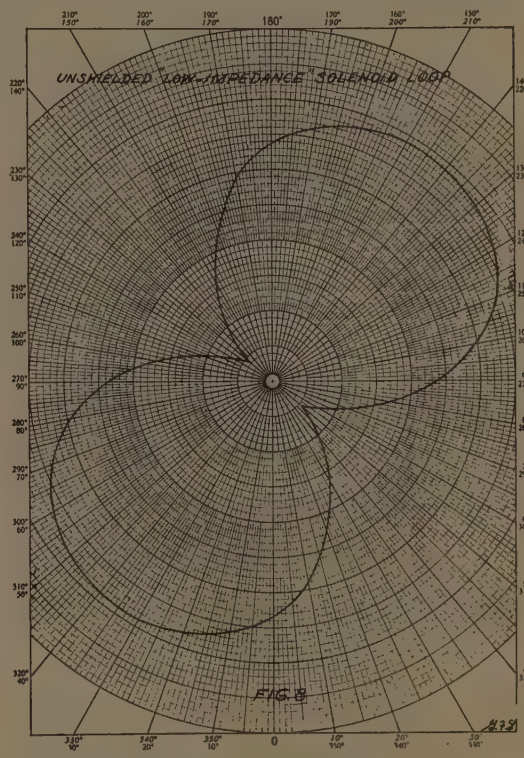

Fig. 8—Unshielded low-impedance solenoid loop antenna.

a given area is minimum for a circular loop because this shape gives the shortest possible perimeter, thereby decreasing the capacitance between turns. This effect can also be reduced by properly spacing the turns to decrease the distributed capacitance. An obvious, though sometimes undesirable, way to decrease the antenna effect, is to construct a loop of small vertical dimensions, but this is often impossible to do since mechanical and other considerations determine the dimensions. Antenna effect may be reduced by electrostatically balancing the winding to ground by means of a center tap. This method, while effective, complicates the mechanical construction and input-circuit details and, therefore, is seldom used. The best and most effective method of eliminating antenna effect is by electrostatically shielding the loop winding. Since the antenna effect is a manifestation of the electrostatic component of the wave, a shield must be used

which greatly attenuates the electrostatic component leaving the electromagnetic component comparatively unaffected. Shields which satisfy this requirement are the tubular gap-type used in low-impedance loops and the Faraday-screen type.

In Fig. 8 is shown the polar characteristic of a typical unshielded low-impedance-solenoid type of loop antenna. While two definite minima are obtained, a complete null is not possible because of a small amount

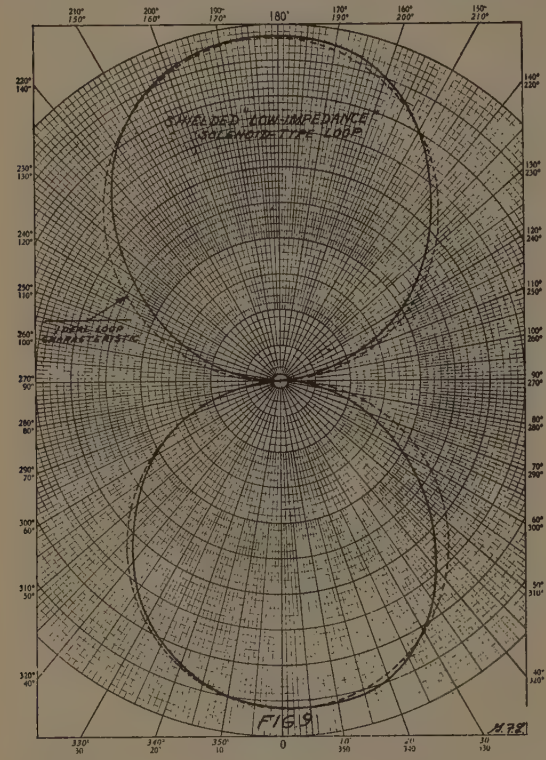

Fig. 9—Shielded low-impedance solenoid loop antenna.

of antenna effect. A very small amount of displacement-current effect is also manifested by unequal magnitudes of the minima. In Fig. 9 is shown the polar characteristic of the same loop in a toroidal gap-type shield. The minima have been reduced to definite "nulls" and the general shape of the characteristic approaches the theoretical double-circle curve very closely. There is, however, a slight difference in the magnitudes of the maxima. The difference in size and position of the curves of Figs. 8 and 9 is due to different scales and has no significance whatever.

In Fig. 10 is shown the polar characteristic of a typical unshielded high-impedance combination solenoid and pancake type of loop of approximately the same diameter as the low-impedance loop just discussed. The minima of this loop are considerably less well defined than the low-impedance type indicating the presence of much more antenna effect and also a very small amount of displacement-current effect. From this curve it is apparent that the antenna effect is also a function of the number of turns. A loop having this characteristic would make a poor direction finder. This polar characteristic, as shown in Fig. 11, can be enormously improved by surrounding the loop with a dome-shaped Faraday type of shield.

The other type of loop winding used is known as the single-pie pancake or spiral winding. The polar pattern of this type is also distorted by two effects,[3] (1) the antenna effect and (2) the winding-pitch effect. Referring to Fig. 12(a), we can see that the displacement-

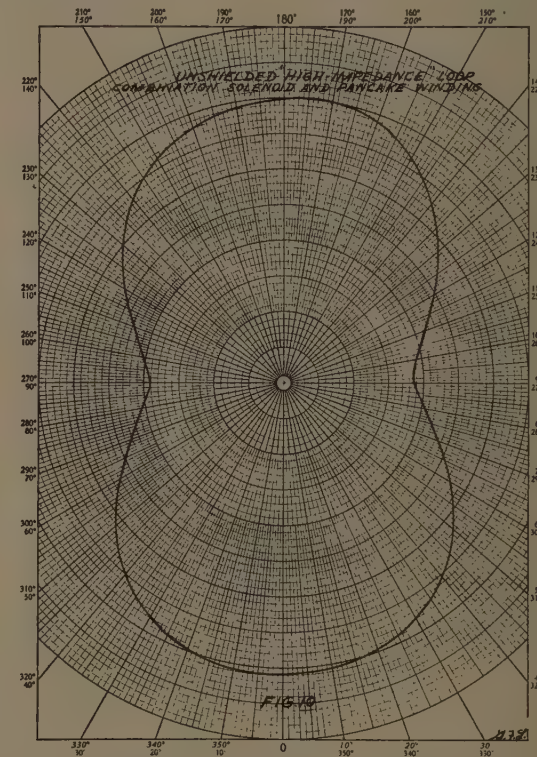

Fig. 10—Unshielded high-impedance loop using a combination solenoid and pancake type of winding.

current effect is entirely absent. If the plane of the loop is oriented at right angles to the radio wave, all the turns lie in the same plane and parallel to the electric field, and there is no phase displacement in the induced voltages of the different turns, and, therefore, no displacement current flows. The antenna effect is much the same as in the solenoid type with the exception that it is of greater magnitude. Blatterman[3] has shown that the antenna effect may be expressed approximately by

$$h + 2ND \tag{22}$$

where h = height of loop
N = number of turns
D = spacing between turns

As in the solenoid type, the polar magnitude is independent of the azimuth angle, but the phase does vary with the angular position of the loop. Blatterman has also shown that the straight loop effect of a pancake loop is composed of two components in quadrature. One of these components is equal to

$$NDE \qquad (23)$$

where E = maximum amplitude of the field intensity in the center of the loop.

The magnitude of this component, also called the winding-pitch effect, is independent of the angular

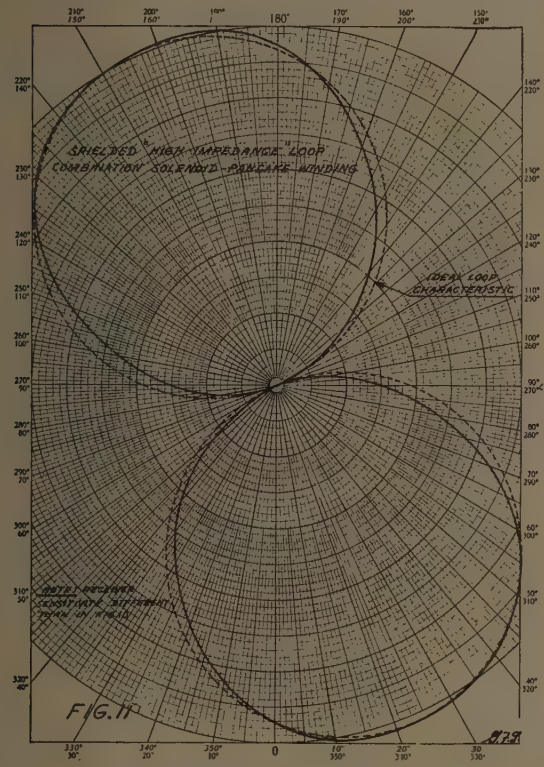

Fig. 11—Shielded high-impedance loop using a combination solenoid and pancake type of winding.

position of the loop, and would appear on the polar diagram as a circle. The other component produces the conventional double-circle polar pattern being equal to the area-turns product times cosine ϕ. Fig. 12(b) shows a polar characteristic of the pancake loop with the various distortive effects. In combining the various curves to obtain the resultant, it must be remembered that the winding-pitch effect is always in quadrature with the conventional-loop effect, and must be added vectorially at every angular position. As already stated, the phase of the antenna effect is a function of the angular position of the loop and must be so considered in order that the three effects may be properly combined to obtain the resultant polar characteristic.

As in the solenoid type of loop, this loop is also to be considered for use as a direction finder; and therefore, it is necesary to obtain sharp minima. The antenna effect may be reduced by the same means already discussed under the solenoid loop while the winding-pitch effect may be reduced by decreasing N and D which in most cases is determined and fixed by space and Q consideration. Electrostatic shielding is the best practical solution to the problem. To illustrate the function

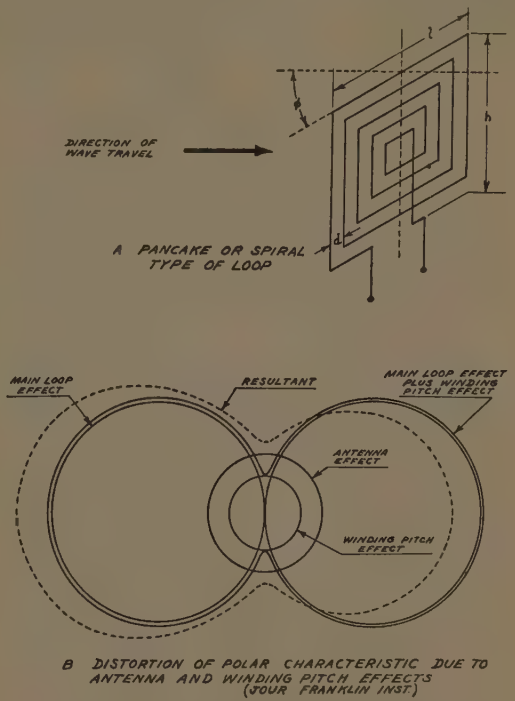
DIRECTION OF WAVE TRAVEL
A PANCAKE OR SPIRAL TYPE OF LOOP
MAIN LOOP EFFECT
RESULTANT
MAIN LOOP EFFECT PLUS WINDING PITCH EFFECT
ANTENNA EFFECT
WINDING PITCH EFFECT
B DISTORTION OF POLAR CHARACTERISTIC DUE TO ANTENNA AND WINDING PITCH EFFECTS (JOUR FRANKLIN INST)

Fig. 12
(a)—Winding diagram of a pancake or spiral type of loop.
(b)—Distortion of polar characteristic due to antenna and winding-pitch effects.

of an electrostatic shield in improving the polar characteristic of a typical range-band high-impedance pancake loop, Fig. 13 shows the characteristic of such a loop having an outer winding diameter of 12 inches and an inner diameter of 5 inches with and without an electrostatic shield. Without a shield the pattern deviates slightly from a true circle, thereby making the loop useless as a direction finder. Surrounding the loop by a dome-shaped symmetrically located Faraday shield promptly restores the conventional double-circle pattern also giving extremely sharp and symmetrically displaced minima. From the data presented, it would seem very desirable, in fact necessary, to shield all direction-finding loops. As a matter of fact, most aircraft loops utilize this type of construction. Besides greatly improving the direction-finding properties of an aircraft loop, an electrostatic shield eliminates certain kinds of electrical noises and effectively reduces the characteristic aircraft precipitation static.

Besides the distortive effects already discussed, there is another effect obtained with both types of loops and known as the quadrantal error. This effect is not inherent in the loops themselves but is a function of the environment in which the loop is placed. On aircraft, the wings and fuselage reflect or refract the radio wave energy causing a distorted wave front. The loop is subjected to this distorted field and erroneous bearings will result if the angular position of the loop relative to the nose of the aircraft is read directly from a uniform azimuth scale. Because these errors occur at 90-

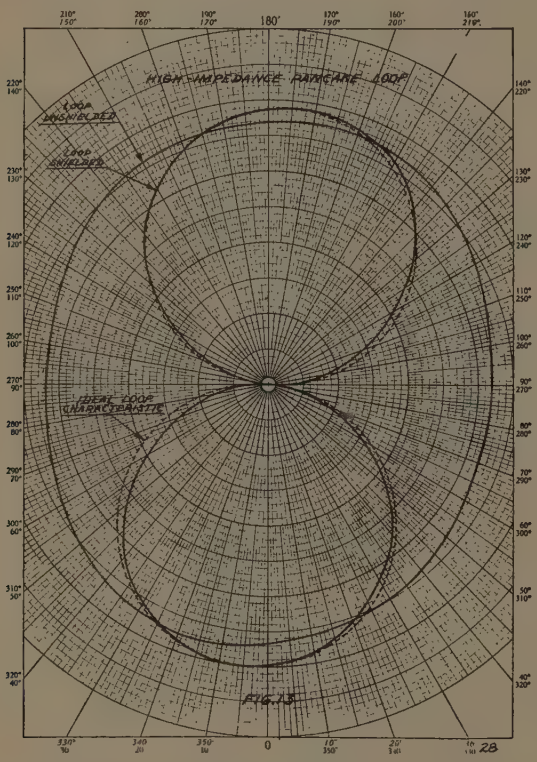

Fig. 13—Shielded and unshielded high-impedance pancake type of loop.

degree intervals, they are known as quadrantal errors. The errors are constant for any one type of aircraft and for any one loop position on the aircraft and can be readily measured and compensated for. It has been found that loops mounted in a nose shell of insulating material give the greatest quadrantal errors, as high as 50 degrees in some cases. The normal installation is either below or on top of the fuselage in which case the quadrantal error may be between 7 to 15 degrees. Of the top and bottom loop mounting positions the bottom position possesses additional navigational advantages over the former.

The effect of the shield on the signal-pickup characteristic of the loop has been investigated somewhat quantitatively, and it has been found that there is a decrease of roughly 2 to 4 decibels which may be attributed to two factors. The Q of the loop is lowered by the addition of the shield, thereby accounting for approximately 30 to 40 per cent of the total reduction. The remaining 60 to 70 per cent may be attributed directly to the shield. The decrease in signal pickup is lower in the low-impedance than in the high-impedance loops because high-impedance loops generally have a much higher initial Q which is influenced considerably by the addition of the shield.

Comparison of Solenoid-Type and Pancake-Type Loop Windings for Aircraft Use

In conclusion, a comparison of the two types of loop windings from the standpoint of aircraft application might be made. The solenoid type of construction adapts itself nicely for low-impedance loops having the toroidal-gap-type shield. A pancake type of winding may also be employed in a low-impedance loop, but it presents somewhat greater constructional difficulties. Very often a combination of the two types of windings is employed as when several shallow pancakes are wound adjacent to each other and connected in series. This type of winding will have a polar characteristic very similar to the normal solenoid type especially when the inner and outer diameters of the winding are almost the same. In the case of low-impedance loops using both types of windings, the Q over the aircraft beacon band of 200 to 400 kilocycles will remain fairly constant, which is desirable.

It has been found in the laboratory that the most practical types of windings to be employed in high-impedance loops are the pancake and the combination winding described above. A straight solenoid type of winding would assume undesirable dimensions. The writer's experience has shown that the combination winding while having a high Q, in the neighborhood of 250, at the low-frequency end has a Q of only 100 or lower at the high-frequency end of the band. This seems to be a property of the type of winding, irrespective of wire size, diameter, and other factors. The pancake type, on the other hand, has a very high and uniform Q characteristic over the entire band. By using an electrostatic shield, a good polar characteristic, together with sharp and symmetrically displaced nulls, can be secured from high-impedance loops employing either types of winding.

The only remaining point of comparison is the signal pickup as measured by the effective height of the two kinds of windings. Assuming loops of the same outside diameter, it is readily seen that the combination winding loop will have a greater area-turns product, which is a measure of the effective height, than the pancake type. To find what the difference actually is, the area-turns product for two types will be compared. In the combination type, since the π's are very shallow, all the turns will be considered to be concentrated at the mean diameter, which although approximate, is sufficiently accurate for practical comparison. Therefore,

we have

$$\text{area} \times \text{turns} = \pi R^2 N. \tag{24}$$

For the pancake type, the area-turns product will be obtained by calculating the area-turns product for one turn at a time and then summing up the individual products to obtain the whole. Referring to Fig. 14 and

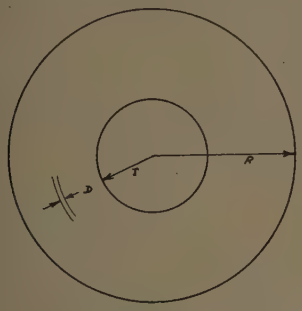

Fig. 14—Diagram for calculation of area×turns of a spiral-wound loop.

letting N equal the number of turns and D, the spacing, we have

N	Area×Turns
1	$\pi(r+0D)^2=r^2\pi$
2	$\pi(r+D)^2=(r^2+2rD+D^2)\pi$
3	$\pi(r+2D)^2=(r^2+4rD+4D^2)\pi$
4	$\pi(r+3D)^2=(r^2+6rD+9D^2)\pi$
5	$\pi(r+4D)^2=(r^2+8rD+16D^2)\pi$
6	$\pi(r+5D)^2=(r^2+10rD+25D^2)\pi$
. .	. . . . .
N	$\pi[r+(N-1)D]^2=[r^2+2(N-1)rD+(N-1)^2D^2]\pi$

The total area-turns product will be the sum of all the terms above or

$$\begin{aligned}\text{area}\times\text{turns}&=\pi Nr^2\\&+2\pi rD[1+2+3+4+\cdots+(N-1)]\\&+\pi D^2[1^2+2^2+3^2+\cdots+(N-1)^2].\end{aligned} \tag{25}$$

The second term is an arithmetic progression the sum of which (for N terms) is

$$2\pi rD\sum(N-1)=\pi rDN(N-1). \tag{26}$$

The third term is the sum of the squares of numbers

$$\pi D^2\sum(N-1)^2=\frac{\pi D^2N}{6}[(N-1)(2N-1)]. \tag{27}$$

Total (area×turns)

$$=\pi\left[Nr^2+rDN(N-1)+\frac{D^2N(N-1)(2N-1)}{6}\right]. \tag{28}$$

In actual loops of this type the number of turns N is about 70 or more so that the expression may be further simplified by considering $N\gg1$.

$$\text{Total (area}\times\text{turns)}=\pi\left[Nr^2+rDN^2+\frac{DN^3}{3}\right]. \tag{29}$$

Area-turns products of actual loops, of the same inductance and outside diameter, using the two different methods of winding have been calculated from the above expressions, and it has been found that for the solenoid or combination type it is about 30 per cent higher than for the pancake type. However, when the effective heights, which are proportional to the area-turns product, are multiplied by the resultant Q, the pancake type actually delivers more signal voltage to the grid of the amplifier by virtue of its higher Q.

Iron-Core Loop Antennas

Iron-core loop antennas have not been used on commercial and military aircraft in this country for several reasons. In the first place, the air-line companies were required by the Government, in the fall of 1937, to install loop antennas on their aircraft for direction-finding and precipitation-static purposes. At that time the iron-core loop was not sufficiently developed for practical use. In the second place the much greater weight of the iron-core loop was, at first, considered a definite disadvantage against its use on aircraft. This disadvantage has since been proved to be erroneous.

As has been shown above, the pickup of a loop-antenna system is dependent on the Q of the loop. The first reason for the introduction of an iron core is to increase the Q in a manner similar to that in a coil. Since the pickup is also dependent on the area-turns product the number of turns used in an iron-core loop will be less by virtue of the permeability of the iron core. Polydoroff[4] has shown that while the product of the Q and the number of turns of an air-core loop and an iron-core loop of the same area and an inductance is the same, the actual signal pickup of the iron-core loop is multiplied by the effective permeability of the iron core. Consequently, for identical signal pickup, the physical size of an iron-core loop can be substantially reduced. It is also possible to mount the loop very close to the metal surface of the aircraft without appreciably lowering the Q. These features simplify the aerodynamic problem. Polydoroff has also shown that antenna or electrostatic effect can be practically eliminated by simply grounding the core. The directional properties of iron-core loops are shown to be considerably improved. This is attributed[4] directly to the higher Q.

An iron core may be used with either the low- or high-impedance type of loop construction with equally good results. However, in the former type a much greater effective permeability can be realized.

[4] W. J. Polydoroff, "Antenna system for wireless communication," U. S. Patent No. 2,266,262, 1941; Great Britain, 1938.

Acknowledgment

The writer wishes to thank P. C. Sandretto, Superintendent of the United Air Lines Communications Laboratory, now on a leave as Major in the U. S. Army, for his encouragement and assistance in the preparation of this paper.

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Deionization Considerations in a Harmonic Generator Employing a Gas-Tube Switch*

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Summary—*A description is given of an experimental investigation of the properties of a thyratron operating as a high-frequency switch in a circuit which permitted the generation of a wide band of harmonics. The experiments indicate that there is an operating frequency below which no difficulties in deionization occur and above which stable operation requires that the grid potential fulfill certain conditions dependent upon the frequency, wave form of the grid voltage, and circuit constants. It has been found possible to operate certain standard thyratrons at switching frequencies as high as several hundred kilocycles per second. For these higher frequencies the deionization of the tubes is incomplete but normal switching behavior is obtained.*

GRID-CONTROLLED gas tubes have received an increasing use in recent years as switching devices. Because of their low impedance, relative lack of inertia, and ability to handle large currents they have many useful applications. Some of the more common are the saw-tooth generator used to supply the sweep for a cathode-ray oscillograph and the submultiple circuit which is used to extend the range of a mechanical counter or to supply a subharmonic of a given frequency. These tubes are also well adapted to the production of current surges of short duration which have a steep wave front. These may be used for a source of harmonics for a carrier-frequency supply or to furnish standard frequencies for calibration purposes. They may also be used for synchronization and triggering of numerous devices which are not necessarily electronic.

All such circuits which use gas-tube switching are eventually limited in the frequency of operation by the time required for the tubes to deionize and to a lesser extent by the ionization time. The following is a description of a series of experiments on a circuit which employs a gas tube as a switch at frequencies well above those at which these tubes have been used in the past. Investigations of the deionization of gas tubes are usually made under what may be called static conditions since the quantity measured is the time required for the tube to regain a certain dielectric strength after a static arc is extinguished.[1,2] In the following experiments the limitations imposed by deionization have been investigated under dynamic conditions and a number of phenomena have been observed which do not appear under static conditions. Many of the phenomena, as would be expected, are the result of the particular tube employed as well as the circuit configuration, but some of the implications are general in character. The results of these investigations have been applied in the use of the circuit as a means of generating a group of harmonics with a relatively high base frequency.

* Decimal classification: R339. Original manuscript received by the Institute, March 11, 1942; revised manuscript received, December 3, 1942. Presented, New York Meeting, April 1, 1942.

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Circuit Characteristics

The general schematic of the circuit investigated is shown in Fig. 1 and the currents and voltages in the

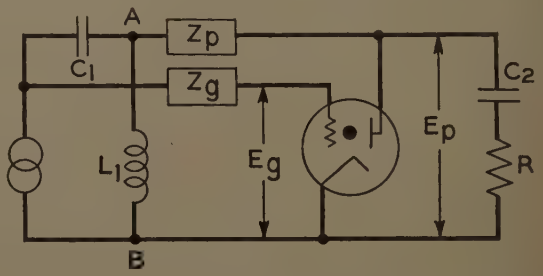

Fig. 1—Harmonic-generator circuit.

[1] A. W. Hull and I. Langmuir, "Control of an arc discharge by means of a grid," *Proc. Nat. Acad. Sci.*, vol. 15, pp. 218–225; March, 1929.

[2] W. E. Berkey and C. E. Haller, "Reignition potential of hot cathode grid glow tubes," *Elec. Jour.*, vol. 31, pp. 483–487; December, 1934.

circuit elements are given in Fig. 2. The inductance L_1 and the capacitance C_1 play the double role of a phase-shifting network and a means of obtaining a large voltage between the points AB. The impedance Z_p, through which the capacitance C_2 is charged, isolates the phase-shifting network from the discharge path during the period that the tube is conducting. Thus it prevents the high harmonics produced from being dissipated in the primary circuit and also limits the primary current which will flow through the tube. The phase of the voltage E_g, appearing between grid and cathode of the gas tube, is adjusted with respect to that of the voltage E_{c2} across the conenser C_2 so that E_{c2} is as nearly at a maximum as possible when E_g intersects the firing characteristic. When the gas tube becomes conducting at this point, C_2 discharges rapidly through the low impedance offered by the tube and the impedance R. The current through R rises sharply at a rate determined by the ionization characteristics of the tube and then falls off approximately exponentially at a rate determined by the time constant RC_2. When C_2 has discharged, the current through the tube will be considerably smaller and the deionization of the space will begin. If a continuous cycle is to be repeated this deionization must have reached a definite limit when the plate potential becomes positive on the succeeding cycle.

Critical Deionization Ratio

When these tubes were operated below a certain frequency limit which depended upon the type of tube and the circuit constants employed, no difficulties with incomplete deionization occurred regardless of the relative amplitude of the grid and plate voltages. In fact, under certain conditions, as will be discussed later, undesirable phenomena result from the occurrence of too-rapid deionization. At higher frequencies, incomplete deionization was manifested by a phenomenon illustrated in Fig. 3 which shows a plot of the potential between plate and cathode as a function of time. On

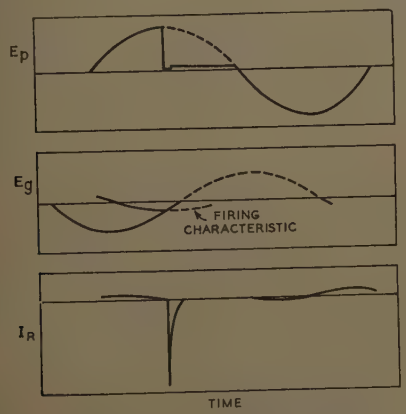

Fig. 2—Voltage E_p between plate and cathode; voltage E_g between grid and cathode; current through the discharge resistance; plotted as functions of time.

successive cycles the tube would break down at different plate-cathode potentials. The number of periods of the fundamental required for a complete cycle of the firing potential depended upon the amplitude

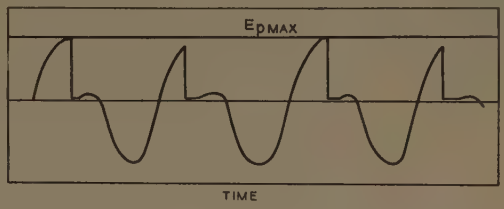

Fig. 3—Appearance of the plate-cathode potential as a function of time for an unstable condition resulting from insufficient deionization.

ratio of the grid-cathode potential to the plate-cathode potential. For values of this ratio above a critical minimum, the operation of the circuit was stable; i.e., the firing potential remained the same from cycle to cycle. For values slightly below the minimum, the tube would fire at some value of plate-cathode potential on one cycle, a lower voltage on the next cycle, and return to the original value the third, etc. For still lower ratios the period of change would last over three cycles and so on for decreasing values of the ratio until finally the tube would break down for every cycle at a low value of the plate-cathode potential.

This deionization phenomenon indicates the production of subharmonics. The process is apparently as follows. Beginning the cycle with the peak breakdown potential, the ionization in the tube after breakdown is sufficiently large so that the tube cannot deionize to its initial state within one cycle so that breakdown occurs at a lower plate potential on the next cycle. This results in a smaller production of positive ions since the charge passed through the tube in the condenser discharge is less. This ionization may be enough less so that on the next cycle the tube is sufficiently deionized to fire at the original value, etc. If not the tube will break down at an intermediate potential on the third cycle, etc. The process may thus be described as a type of hunting.

The fact that the grid swing has a pronounced effect on this phenomenon confirms what had been known previously—that the grid plays a major role in the deionization of the tube. The question as to whether the plate performed any function in deionizing the tube was established by determining the minimum grid swing to maintain stable operation as a function of the plate swing, in one case when the plate was allowed to swing negative and in the other case when it was not, all other conditions remaining the same. In order to prevent the plate from swinging negative, a diode was connected between the plate and cathode with its polarity such that it conducted when the plate tended to swing negative and kept the plate-cathode voltage small. The potentials between grid and cathode and plate and cathode were measured

from oscillograms. The grid voltage was phased as nearly as possible 90 degrees lagging the plate voltage. The results of these measurements are shown in Fig. 4. E_p represents the value of the plate-cathode voltage at the instant the tube fired, which corresponds very

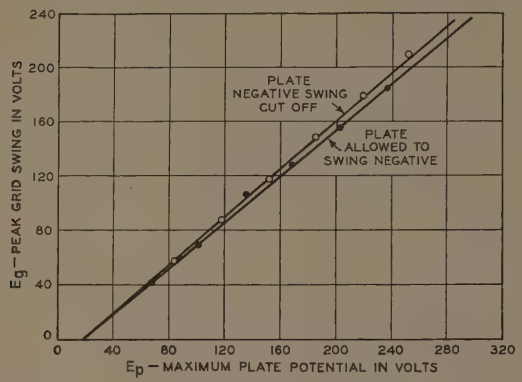

Fig. 4—Minimum grid swing to maintain stable operation as a function of the plate potential at firing. In one case the plate was permitted to swing negative and in the other it was prevented by a diode from going more than a few volts negative. Switching frequency, 60 kilocycles per second. Western Electric 338A tube.

nearly to the peak applied voltage. The quantity actually of interest is the condenser potential at the instant of breakdown. However, the impedance of the condenser to the fundamental is very much higher than that of discharge resistance R, and hence the potentials across the tube and the condenser are prac-

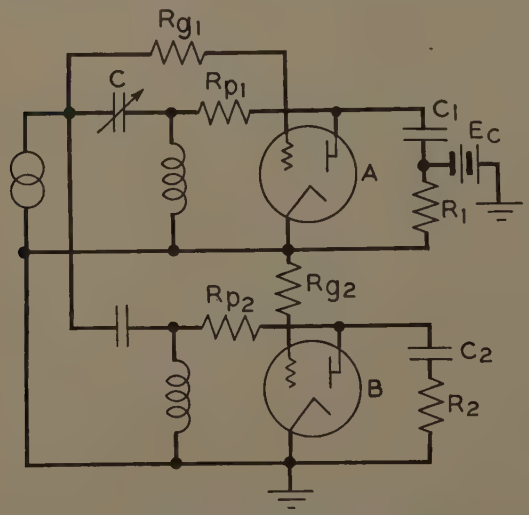

Fig. 5—Harmonic-generator circuit employed in studying the effect of triggered operation upon the deionizing conditions.

tically identical prior to breakdown. The value of E_g was the peak negative value of the grid swing relative to the cathode as measured directly at the grid. Fig. 4 indicates that the plate has only a small deionizing action. This is an interesting observation since it indicates that the important region for deionization is that which is screened from the plate by the control grid and hence in designing gas tubes for high-frequency operation it is this region which should be constructed to favor deionization.

In contrast to the minor function of the plate in deionizing the tube the following experiment indicates the importance of the grid in this respect. A comparison was made of the minimum grid swing necessary to maintain proper operation as a function of the plate swing under two conditions. The first involved sinusoidal operation of the circuit of Fig. 1, and the second operation with the circuit of Fig. 5. In this circuit tube B is under test. The circuit employed in operating tube B differs from that of Fig. 1 in that the grid bias is fixed until the tube A, which is operated in the circuit of Fig. 1, fires. When this occurs a sharp voltage pulse is applied to the grid of tube B. By adjusting the condenser C, the phase of the voltage pulse produced by tube A could be adjusted with respect to the plate-

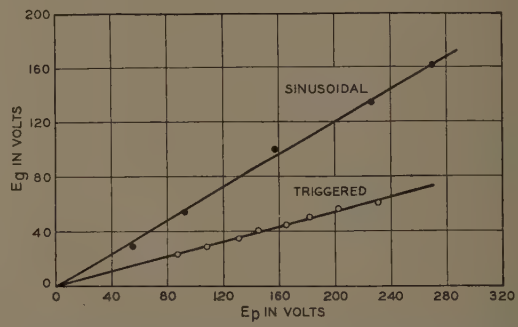

Fig. 6—A comparison of the minimum amplitude of grid swing to maintain stable operation as a function of the plate swing for the case of sinusoidal and triggered operation of the control grid. Western Electric 338A tube. Switching frequency, 60 kilocycles per second, $1/RC=66$ megacycles per second.

cathode swing applied to tube B. The phase adjustment was made so that tube B triggered at the maximum value of this swing. In this case the voltage E_p has the same significance as before while the voltage E_g refers to the value of the bias E_c. The grid resistance was necessarily made small in order to avoid distortion of the discharge wave. Hence this voltage was practically the same at any instant, except during the actual triggering, as the voltage at the grid. The results are shown in Fig. 6. The marked change in the critical ratio is in contrast to the slight change produced in the foregoing experiment. The fact that the critical ratio is smaller for triggered than for sinusoidal operation is in the expected direction since for triggered operation the grid is negative for a larger proportion of the cycle. Also since the grid is held negative until the instant of firing it can more readily maintain an overlapping ion sheath about the grid opening than in the sinusoidal case.

Fig. 7 shows the variation of the critical ratio with frequency. One very interesting feature of these curves is that there is apparently an intercept frequency below

which the ions need not be drawn out by a field on the grid. Below this threshold frequency the drift of the ions to the elements and walls resulting from other causes completes the deionization. The threshold frequency would be expected to be a function of the ion

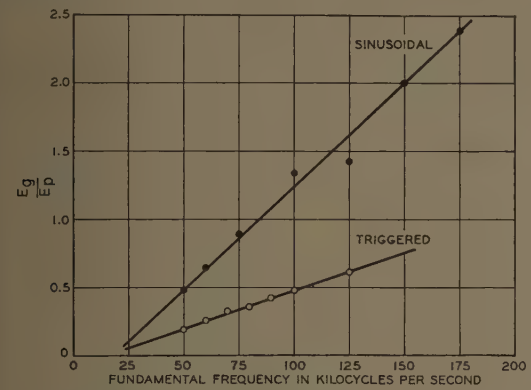

Fig. 7—The critical ratio of the amplitudes of grid to plate potentials as a function of frequency for triggered and sinusoidal grid control. Western Electric 338A tube. $1/RC = 66$ megacycles per second.

density after the discharge or in other words to depend upon the discharge constants. The fact that no critical ratio for stable operation existed at low frequencies was verified, i.e., at frequencies below the critical values indicated by Fig. 7, the tube would ionize at the same values of the plate voltage on successive cycles irrespective of the ratio of the grid and plate swings. The data given are all for a Western Electric 338A tube. A similar behavior was found for the Western Electric types 287A and 323A which differ in structure and gas-filling from the 338A.

Surge Phenomena

The experiments concerning the effect of frequency on the stability ratio indicated that some other source

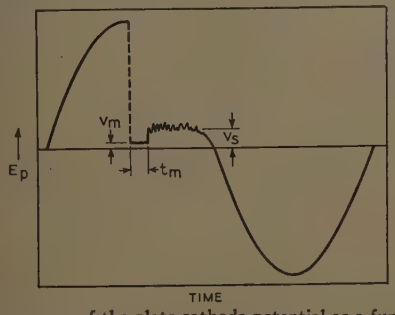

Fig. 8—Appearance of the plate-cathode potential as a function of time for low frequencies.

of deionization existed in addition to that caused by the negative grid. This source reduces the ionization below a critical minimum for frequencies lower than a limit which depends upon the circuit constants. The following experiments which were carried out at a frequency below the critical value shed additional light on this phenomenon.

An oscillographic trace of the plate-cathode potential as a function of time for the circuit of Fig. 1 appeared as shown in Fig. 8 for lower frequencies. The condenser discharge except for the longest surge durations involved in this investigation was completed during the fall of the plate-cathode potential. When the arc struck, the plate-cathode potential fell to a value

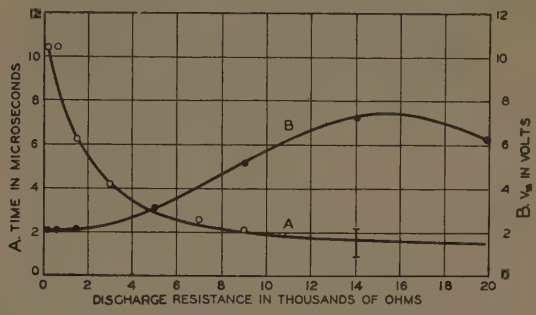

Fig. 9—The effect of discharge resistance upon the minimum arc drop V_m and the time t_m to the potential jump. Western Electric 338A tube. Switching frequency, 4 kilocycles per second, $C_2 = 25 \times 10^{-12}$ farad, $R_p = 10{,}000$ ohms.

considerably below that which is characteristic of a static discharge. After a period of time determined by the circuit conditions the potential rose again to a value which, on the average, was approximately characteristic of the steady discharge. The arc potential after the rise was, however, not constant but oscillated about the static value. The character of these oscillations was found to depend upon the tube type and the discharge constants. An oscillographic investigation was made of the minimum arc drop V_m and the time interval t_m, between breakdown to the minimum potential and the rise to the static arc drop as a function of the circuit constants. The frequency at which these studies were made was 4 kilocycles per second and the

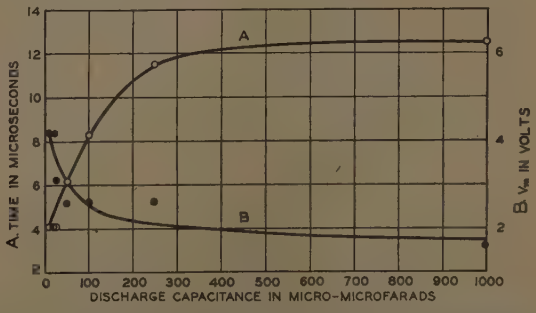

Fig. 10—The effect of discharge capacitance upon the minimum arc drop and the time to the potential jump. Western Electric 338 A tube. Switching frequency, 4 kilocycles per second, $R = 600$ ohms, $R_p = 10{,}000$ ohms.

tube employed was a Western Electric 338A. The grid and plate potentials were held fixed and phased so that the tube triggered at the peak of the voltage wave.

Fig. 9 shows the effect of the discharge resistance upon the time t_m and the minimum arc drop. Fig. 10

shows the effect of the discharge capacitance and Fig. 11 the effect of the charging resistance $Z_p = R_p$. In the latter case the variation of V_m was less than the accuracy of measurement.

A simple explanation may be made for these phenomena. When the tube breaks down a large current is passed which requires the formation of a high ion density in the discharge path. After the condenser discharge is complete, current requirements for the tube are dictated by the charging resistance R_p, and the applied potential and normally this current is much smaller than the condenser-discharge surge. The positive ions because of their large mass do not move ap-

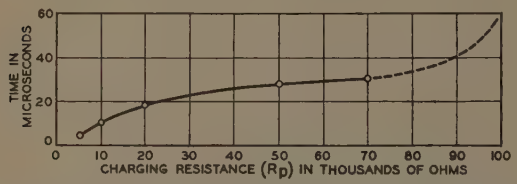

Fig. 11—The effect of the charging resistance R_p upon the time to the potential jump. Switching frequency, 4 kilocycles per second, $C_2 = 100 \times 10^{-12}$ farad, $R = 300$ ohms. Western Electric 338A tube.

preciably from the positions at which they are formed during the major discharge so that immediately afterward there exists a sort of frozen positive space charge. This permits the flow of electrons required by the charging current with very low potentials applied across the tube. When this positive space charge has been sufficiently depleted by drift to the electrodes the potential across the tube begins to rise in order to form new ions. If the rate at which the positive ions are removed from the space occurs more rapidly than the potential can build up on the discharge condenser the tube potential may rise above the static arc drop. It may then continue to build up until a dynamic firing point is reached when breakdown will again occur producing a smaller surge of positive ions. This cycle repeated offers an explanation of the oscillations about the firing characteristic.

The data presented in Figs. 9 to 11 are in agreement with this explanation as inspection will show. Thus, for a fixed discharge capacitance, the peak current during the condenser discharge will decrease as the discharge resistance is increased and the ion density will be reduced. Hence the minimum arc drop required to pass the current from the primary circuit in the period following the discharge will rise and the time before the positive-ion density is depleted sufficiently to require renewed production will decrease. Similarly as the discharge capacitance is increased with the other constants fixed, the total charge passed through the tube will increase and the space should be more completely filled with a positive space charge. Hence for increasing values of C it would be expected that the value of V_m would decrease and t_m would increase. Finally, as the resistance R_p is increased the current through the tube after the condenser discharge will be reduced. As a result, the potential V_m, necessary to maintain this current, will be smaller so that the positive-ion losses caused by the drift in this field will be lessened. The result is an increase in the value of t_m and a decrease in V_m. The latter was found to occur but for the circuit constants employed for this case the variation was within the accuracy of measurement and is not shown.

As the frequency of the fundamental was increased a value was reached which depended upon the circuit constants at which the potential rise across the tube did not appear. These frequencies were of the same order as those at which a critical ratio of the grid and plate swings first appeared and it seems probable that these phenomena are related. Thus, if sufficient deionization does not occur in the interval immediately following the discharge while the plate is positive, it must be supplemented by the deionization caused by a drawing out potential on the grid.

A similar behavior was found for another type of tube, the Western Electric 287A, when the same studies were made.

The phenomena just described are fundamental to the operation of one familiar gas-tube-switching device, the saw-tooth sweep generator. An observation of the voltage across the gas tube in this case will show the same very low arc drop following the return sweep. Operation of the sweep will be stable if at the termination of the time interval t_m, the recharging of the sweep condenser from the minimum arc drop proceeds sufficiently slowly so that the tube recovers its dielectric strength through positive-ion losses before the potential rises to the point where renewed ion production can begin. This explanation which does not depend on the common explanation of residual inductance in the leads has been advanced by Drewell.[3] It is of interest to note that when a sweep circuit fails at its upper-frequency limit, the failure occurs in the same manner as for the harmonic-generator circuit; i.e., the amplitude of the sweep on successive cycles will differ.

Discussion

The foregoing results show that for operation in this circuit there are two sources of deionization. Immediately after the discharge while the plate potential is positive there is a large excess of positive ions. These are removed in part by the discharge between plate and cathode. When the plate potential swings negative, the deionization effected by the plate is small and the remaining deionization is principally caused by the negative grid.

It is interesting to note that most of the phenomena can be explained on the basis of three assumptions:

(1) The number of ions formed is proportional to the peak voltage at breakdown, $I = KE_p$.

[3] P. Drewell, "Die Wirkungsweise der gittergesteurten Gasentladungsröhren usw.," *Zeit. für Tech. Phys.*, vol. 17, pp. 249–262; 1936.

(2) The rate of deionization while the plate is positive is proportional to the instantaneous applied plate potential,

$$\frac{dI_1}{dt} = -K_1 E_p \sin \omega t$$

where I_1 is the total number of ions removed per cycle during the interval that the plate-cathode potential is positive.

(3) The deionization by a negative grid is proportional to the instantaneous grid potential,

$$\frac{dI_2}{dt} = -K_2 E_{gs} \sin \omega t.$$

Here I_2 is the total number of ions removed per cycle by the negative grid. With these assumptions one may derive the following equations.

The decrease of ionization casued by the residual plate current will occur during a quarter period following the condenser discharge.

Thus,

$$I_1 = -\int_{1/4f}^{1/2f} K_1 E_p \sin \omega t dt = -\frac{K_1 E_p}{\omega}.$$

The decrease of ionization caused by the grid will occur over approximately a half cycle.

Thus,

$$I_2 = -\int_{1/2f}^{1/f} K_2 E_{gs} \sin \omega t dt = -\frac{2K_2 E_{gs}}{\omega}.$$

Then, for stability,

$$KE_p = \frac{K_1 E_p}{\omega} + \frac{2K_2 E_{gs}}{\omega}$$

or

$$\frac{E_{gs}}{E_p} = \frac{1}{2K_2}[K\omega - K_1].$$

For any given frequency this shows that the ratio of grid to plate swing is independent of amplitude in agreement with the experimental results of Fig. 6. It also predicts that the ratio will increase linearly with frequency and intercept the frequency axis at a positive value in agreement with the results of Fig. 7. A corresponding analysis for triggered operation yields

$$\frac{E_{gT}}{E_p} = \frac{1}{2\pi}\left[\frac{K'\omega}{K_2} - \frac{K_1}{K_2}\right].$$

K' differs from K since the quantity of ionization which must be removed to maintain stability for triggered and sinusoidal operation may differ. This is apparent since for triggered operation the grid voltage is strongly negative until the instant of triggering and hence can maintain an overlapping ion sheath in the presence of a higher ion density than is the case for sinusoidal operation where the grid voltage decreases until the instant of firing. For this reason one would expect K' would be less than K.

Thus upon comparing the stability ratios for triggered and sinusoidal operation

$$\frac{\dfrac{E_{gs}}{E_p}}{\dfrac{E_{gT}}{E_p}} = \pi \frac{\dfrac{K\omega}{K_1} - 1}{\dfrac{K'\omega}{K_1} - 1}.$$

Now from the condition that the critical ratio shall be zero one obtains

(a) for sinusoidal operation

$$\omega_0 = \frac{K_1}{K}$$

(b) for triggered operation

$$\omega_0' = \frac{K_1}{K'}.$$

Thus,

$$\frac{\dfrac{E_{gs}}{E_p}}{\dfrac{E_{gT}}{E_p}} = \pi \frac{\dfrac{\omega}{\omega_0} - 1}{\dfrac{\omega}{\omega_0'} - 1}.$$

The values for the frequencies of zero critical ratio may be obtained from the data of Fig. 7. Then substituting these values and the frequency of 60 kilocycles per second at which the data of Fig. 6 were measured

$$\frac{E_{gs}}{E_p} \Big/ \frac{E_{gT}}{E_p} = 2.3.$$

This is to be compared with an experimental value of 2.2 from the data of Fig. 6. The data of Fig. 7 were obtained in an independent run. Substituting the values of the critical ratio at 60 kilocycles per second for sinusoidal and triggered operation, a value of 2.48 is obtained. Thus agreement is obtained within the experimental error of the measurements.

We have obtained, therefore, on the basis of the assumptions noted, qualitative agreement with the experimental variation of the ratio with frequency and amplitude and more remarkably (considering the arbitrariness of the assumptions), quantitative agreement with the effect of the wave form of the grid potential on the critical ratio. The assumptions made are valid in part only under the restricted conditions of the particular mode of operation employed. The proportionality constants will depend on the circuit constants of the circuit. For example one would expect the constant K to involve the discharge capacitance C, and although a complete investigation was not made, somewhat limited data indicate that the critical ratio for a given frequency is proportional to the discharge capacitance. The author does not intend to press the foregoing speculation too strongly since the basic assumptions are largely empirical. Because of the

complexity of the processes involved and the restricted nature of the experimental observations, a more fundamental analysis into the basic assumptions has not been fruitful.

Dynamic Control Characteristics

The deionization times of 100 to 1000 microseconds which are commonly given for gas-filled tubes may give rise to the erroneous impression that such tubes

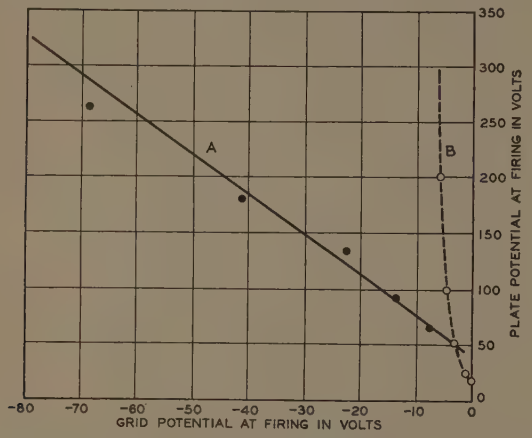

Fig. 12—A comparison of *A* the dynamic and *B* the static firing characteristics of a gas tube. In case *A* the grid swing was held at the minimum necessary to maintain stable operation. Switching frequency, 60 kilocycles per second. Western Electric 338A tube, $R=300$ ohms, $C_2=25\times10^{-12}$ farad.

are not operative at frequencies where switching is carried out at frequencies above the reciprocal of these times. This circuit was operated above such frequencies and the following experiments show that gas-tube switching may be carried out successfully although the state of deionization of the tubes is far from complete.

Fig. 12 shows a dynamic firing characteristic obtained when the ratio of the grid and plate swings was held at the critical value. For comparison the static characteristic, curve *B*, is shown. The considerably more negative grid values at firing in the dynamic case indicate that the tube is incompletely deionized. Fig. 13 shows the effect of the grid swing on the firing characteristic. In this case for a fixed frequency and a fixed value of the plate potential at which breakdown occurred the grid potential at firing was determined as a function of the grid swing. These results show, as Berkey and Haller[2] have pointed out, that the deionization time for a gas tube is not a definite quantity and further, that the firing characteristics given for gas tubes are significant only under specific conditions and in particular for operation at low frequencies. At higher frequencies the firing characteristic will be dependent upon the wave form of the grid voltage and upon the discharge constants.

Harmonic Generator

The various phenomena which have been discussed above were studied in connection with an investigation of the operation of the foregoing circuit as a harmonic generator. The circuit was suggested by Peterson as an electronic analogy to a magnetic harmonic generator which has been described in the literature.[4] The magnetic generator has some considerable advantages from the standpoint of efficiency particularly at high frequencies and also in power capabilities, but the tube circuit is somewhat more flexible and also the gas-tube-switching element is readily available commercially. One further advantage in the tube circuit is that the switching element is free of inductance which permits discharges having steeper wave fronts. The last statement requires some qualification when the duration of the discharges becomes comparable with the ionization time of the tube. In this case, as Manley has shown in some unpublished results, the tube behaves as though it contained a very small inductance. The inductance depends upon the method of firing the tube. There are many applications for harmonic generators where efficiency and power are not major considerations, such as standard-frequency sources, and in some of these the gas-tube harmonic generator may be preferable to the magnetic generator.

A number of hot-cathode gas tubes were tested in this circuit. The most suitable tried were found to be the Western Electric types 338A, 287A, and 323A. The 338A tube has a grid coaxial with the cathode. A small single circular opening is provided in the grid and the anode is provided by a heavy wire parallel to the cathode and on a radius through the grid opening. Only a small fraction of the active cathode "sees" the

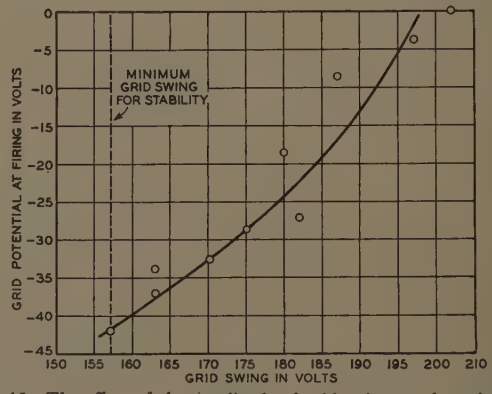

Fig. 13—The effect of the amplitude of grid swing on the value of the grid potential at firing for a fixed plate swing. Western Electric 338A tube. Switching frequency, 60 kilocycles per second, $R=300$ ohms, $C=25\times10^{-12}$ farad.

anode directly. The tube has an argon filling. The structure of the 287A and 323A is the same. The cathode is a multifold mesh ribbon, which forms a rectangular-shaped cathode with the ribbon edges facing the grid opening. The grid is a cylindrical structure with the end facing the anode closed except for a

[4] E. Peterson, J. M. Manley, and L. R. Wrathall, "Magnetic generation of a group of harmonics," *Jour. A. I. E. E.*, vol. 56, pp. 995–1001; August, 1937.

single opening through which the discharge passes to the anode which is a circular concave disk parallel to the cylindrical face of the grid. The cylindrical portion of the grid tends to inhibit a discharge around the flat end. The 287A is a mercury-vapor tube. The 323A has a mercury filling and in addition contains argon gas which tends to eliminate the dependence of the characteristics on temperature. All three tubes have the common characteristic that the discharge is restricted to a single opening in the grid structure. Tubes with mesh or wound grids in general would not produce as sharp a breakdown because of their longer ionization time. It was found that under some circumstances a sharp breakdown was obtained with a tube having a mesh or wound type of grid. When this occurred the discharge concentrated apparently through a more open region of the grid. The behavior, however, was erratic under these circumstances since the discharge tended to shift to various points on the grid.

Fig. 14 shows a reproduction of oscillographic tracings of current surges produced in this circuit. These were observed by means of a synchronized sinusoidal megacycle sweep circuit whose full amplitude is not shown. The fundamental frequency of operation for these observations was 60 kilocycles per second. The circuit constants were the same in each

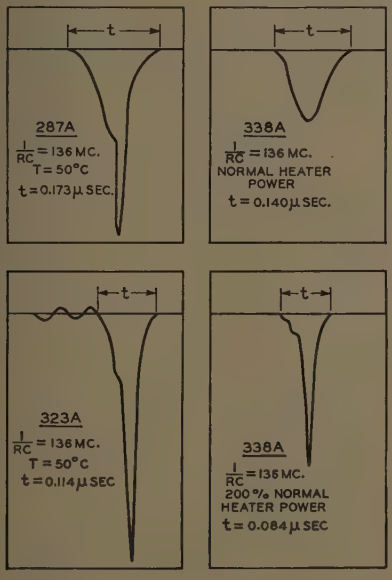

Fig. 14—Oscillographic tracings of the potential across the discharge resistance for three different types of tubes with the same discharge constants. The effect of increased emission for the Western Electric 338A tube is also shown. The frequency of switching was 60 kilocycles per second, $R = 377$ ohms, $C = 20$ micromicrofarads.

case. The reciprocal time constant of the discharge circuit was 132 megacycles per second. The durations of the discharges are considerably shorter than the values usually given for gas-tube circuits and this may be ascribed mainly to the use of a tube with a single discharge opening and the method of operation which favors rapid ionization. It is of interest to note the effect of an increased cathode emission from the 338A tube. The peak current for the tube in each case was considerably less than the maximum rated current.

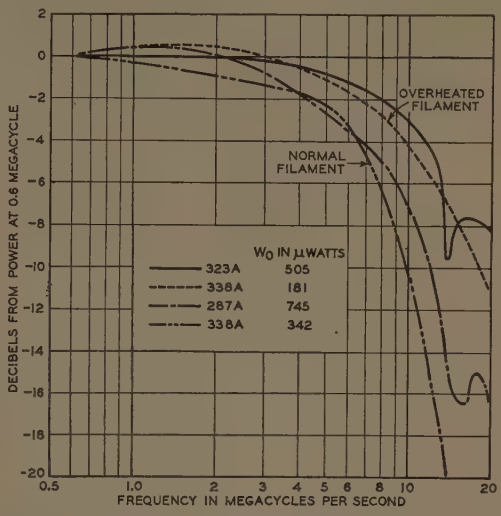

Fig. 15—The distribution of energy over the harmonic spectrum corresponding to the pulses shown in Fig. 14.

The cathode-grid structure is such, however, that for surges of as short a duration as those involved here the emission is effective for only that portion of the cathode immediately before the grid opening since the ionization formed in this region does not have sufficient time to diffuse to the more remote regions and neutralize the space charge before the condenser discharge is complete. The 287A and 323A tubes are better adapted in this respect.

Fig. 15 shows the distribution of energy in the harmonic spectrum with reference to the lowest harmonic measured. The current analyzer which was employed did not permit measurements below the tenth harmonic or 600 kilocycles per second. The value for the power at this frequency is denoted as W_0. If the discharge were of an exponential form

$$i = I_0 \varepsilon^{-t/RC_2}$$

the energy in this spectrum would decrease slowly with the harmonic order and should have decreased to a level 3 decibels below that for the lowest harmonic at a frequency $1/2\pi RC_2$. It will be observed that for surge durations as short as those illustrated the distribution differs from the predicted result. The deviations result from the departure from the ideal condenser-resistance discharge form caused by the ionization time of the tube, and the resistance of the gas path and also from the difficulty in eliminating small lead inductances.

Effect of Vapor Pressure

The vapor pressure for a gas-filled tube has a marked effect on the course of the breakdown as would be expected. This is shown in Fig. 16 where a number of oscillogram tracings for a given set of discharge constants and various bulb temperatures are given. The more rapid breakdown at higher pressures is the result of the increased probability of collison. A limitation

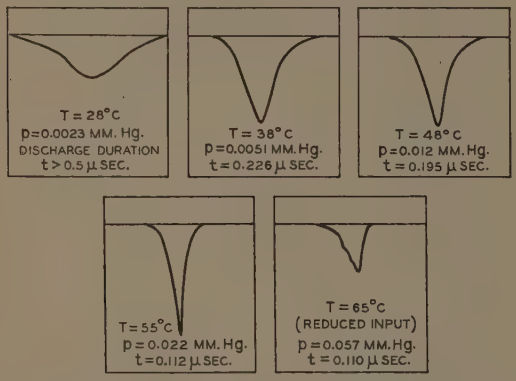

Fig. 16—The effect of vapor pressure on the pulse form for a Western Electric 287A tube. The switching frequency was 60 kilocycles per second.

in the improvement which may be effected by an increase of pressure results from difficulty in maintaining stable operation in the sense discussed earlier, since the critical ratio increases rapidly with pressure. Because of the effect of temperature upon vapor pressure, it is desirable to use tubes filled with a rare gas. However, a tube of the 323A type which has a combination filling of mercury and a rare gas operates satisfactorily.

Operating Frequencies

This circuit has been operated at fundamental frequencies as high as 300 kilocycles with standard tubes. The operating efficiency at such frequencies is low because of the large potentials required on the grid circuit to maintain stable operation. The inclusion of a large resistance in the grid circuit is not effective since it requires a corresponding increase in the applied potential to maintain stability. However, the circuit modification shown in Fig. 17 should permit improved operation. In this case a diode is connected in series with the grid and poled so that the grid cannot be driven positive through the diode. In shunt with the diode is connected a high resistance to avoid charging difficulties. This permits a low resistance in the grid circuit during deionization and a high resistance during the period when the driving voltage is positive.

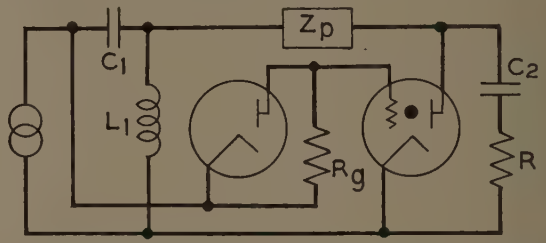

Fig. 17—A circuit schematic for a harmonic generator illustrating the use of a diode to prevent power loss in the grid circuit during the positive excursion of the grid-control potential.

Acknowledgment

The author wishes to express his appreciation to Dr. Eugene Peterson of these Laboratories under whose guidance this work was conducted, for much helpful advice in the experimental work, and for suggestions in the preparation of the manuscript. He is also greatly indebted to Mr. L. G. Kersta whose early experiments with the circuit are intimately bound up with the work here described.

A Note on the Characteristics of the Two-Antenna Array*

CHARLES W. HARRISON, JR.†, ASSOCIATE, I.R.E.

Summary—*The definition for the "effective length" of a transmitting antenna, which was recently brought to the attention of* PROCEEDINGS *readers, is used in deriving expressions for the radiation function, radiation resistance, directivity, and gain of a two-vertical-antenna array, when the relative phase of excitation and current amplitudes are of arbitrary value. The formulas derived for computation of the array characteristics are so simple that they may be applied by anyone having a rudimentary knowledge of antennas and mathematics. Since the assumption of a perfect earth is made, the results are precise only for antennas operated at broadcast frequencies, and at longer wavelengths.*

NUMEROUS articles have appeared in the literature dealing with the subject of directivity patterns, radiation resistance, and other characteristics of antenna arrays.[1–11] Without doubt, the problem of the two-vertical-antenna array, when the relative phase and current amplitudes have any given value, can be solved using methods already described. It is thought, however, that with the introduction of a new definition for the "effective length" of a transmitting antenna, a re-examination of the solution of certain antenna problems is warranted, particularly in view of the great simplification usually obtained. It is with this thought in mind that the writer has prepared the present paper.

The effective length of a transmitting antenna as given by King[12] is defined as the coefficient of the leading term in a Fourier expansion of the field characteristic, for antennas of half length h which are less than λ/π, but not for longer ones. More precisely,[13]

$$V(\theta) = \frac{\cos(H\cos\theta) - \cos H}{\sin\theta \sin H} \doteq \frac{2(J_0(H) - \cos H)\sin\theta}{\sin H} \doteq H_e \sin\theta. \qquad (1)$$

Here $V(\theta)$ is defined as the field characteristic referred to the input current.

$H = \beta_0 h$, $\beta_0 = 2\pi/\lambda$; h is the antenna half length or the full length for an antenna located over a perfectly conducting half space. The latter definition for h is applicable in this analysis.

θ is measured from the vertical, for linear radiators, lying parallel to the z axis, as shown in Fig. 1.

H_e is the effective length of the antenna. It is to be borne in mind that

$$V(\theta) \doteq H_e \sin\theta \qquad (2)$$

only when

$$0 \leqq H \leqq 2. \qquad (3)$$

For longer antennas, the second term in the Fourier expansion for $V(\theta)$ will be required.[14]

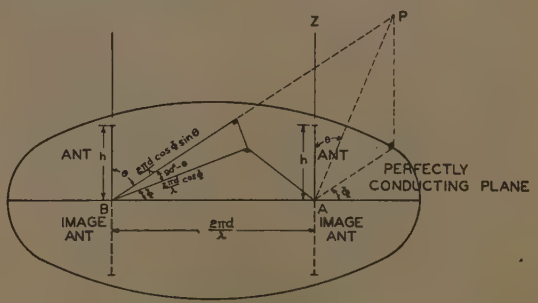

Fig. 1—The two-antenna array.

Note: Point P is the distant zone. If I_B leads I_A by γ, then choose I_A as the reference current, and $I_B = KI_A e^{+j\gamma}$. If I_A is to be used as the reference current, I_B should not lead I_A by more than 180 degrees.

Although the derivation of $V(\theta)$ is based on the assumption of a sine current distribution, one advantage of the approximate representation of the field characteristic is that the radiation resistances so determined from its use agree more closely with the

* Decimal classification: R125. Original manuscript received by the Institute, April 27, 1942.

† Cruft Laboratory, Harvard University, Cambridge, Massachusetts.

[1] Ronald M. Foster, "Directive diagrams of antenna arrays," *Bell Sys. Tech. Jour.*, vol. 5, p. 292; April 1926.

[2] G. C. Southworth, "Certain factors affecting the gain of directive antennas," PROC. I.R.E., vol. 18, pp. 1502–1537; September, 1930.

[3] E. J. Sterba, "Theoretical and practical aspects of directional transmitting systems," PROC. I.R.E., vol. 19, pp. 1184–1216; July, 1931.

[4] P. S. Carter, C. W. Hansell, and N. E. Lindenblad, "Development of directive transmitting antennas by R.C.A. Communications, Inc.," PROC. I.R.E., vol. 19, pp. 1773–1843; October, 1931.

[5] P. S. Carter, "Circuit relations in radiating systems," PROC. I.R.E., vol. 20, pp. 1004–1042; June, 1932.

[6] Stuart Ballantine, "On the radiation resistance of a simple vertical antenna at wave lengths below the fundamental," PROC. I.R.E., vol. 12, pp. 823–833; December, 1924.

[7] W. W. Hansen and J. G. Beckerly, "Concerning new methods of calculating radiation resistance either with or without ground," PROC. I.R.E., vol. 24, pp. 1594–1622; December, 1936.

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[9] G. H. Brown and R. W. P. King, "High-frequency models in antenna investigations," PROC. I.R.E., vol. 22, pp. 457–481; April, 1934.

[10] G. H. Brown, "Directional antennas," PROC. I.R.E., vol. 25, pp. 78–145; January, 1937.

[11] S. A. Schelkunoff, "A general radiation formula," PROC. I.R.E., vol. 27, pp. 660–666; October, 1939.

[12] Ronold King, "Distant field of linear radiators," PROC. I.R.E., vol. 29, pp. 458–464; August, 1941.

The following corrections to this paper have been approved by Dr. King: In (3), write B for R; in (9), write β for 0 in the exponent; in (12) $I_r I_r^*$ should not be squared; in (16), $(4\pi R_0\beta/\pi)$ should be squared; in (27), the subscript on R^e should be o instead of m. In the third line below (27), the superscript bar should be omitted from the second CiX. In (73), (81), (82), and (86), the factor $\sin H$ or $\sin^2 H$ in the denominator should be omitted. In the eighth line from the end of the article, (53) and (52) should be interchanged.

[13] See equations (52), (53), and (61) of footnote 12.

[14] See equations (54) and (62) of footnote 12.

values obtained from the rigorous analysis, when the assumption of an infinitely thin conducting thread is not made.[15,16]

Now it can be shown that a dimensionless radiation function referred to the input current for any vertical-antenna array, when erected over a perfectly conducting half space can be defined by

$$K_0^2(\theta, \Phi) = 4V^2(\theta)P^2(\theta, \Phi). \tag{4}$$

The quantity

$$R_0^e = \frac{15}{4\pi}\int_0^{2\pi}\int_0^{\pi} K_0^2(\theta, \Phi)\sin\theta d\theta d\Phi, \tag{5}$$

is the radiation resistance in ohms referred to the input current in either antenna.

The directivity of the array, corresponding to that given by Carter, Hansell, and Lindenblad[17] is

$$D = \frac{15K_0^2(\theta_m, \Phi_m)}{R_0^e}. \tag{6}$$

Here θ_m and Φ_m are the values of θ and Φ corresponding to the direction of maximum radiation. For the problem under discussion, $\theta_m = \pi/2$ under all conditions.

The gain of an antenna array, expressed in decibels, is defined by

$$\text{db gain} = 10\log_{10}\frac{D}{D_0} \tag{7}$$

where D_0 is the directivity of the reference antenna, which may be taken conveniently as a half-wave dipole in free space. Such a dipole has a directivity D_0 of 1.64, if one assumes an infinitely thin conducting thread.

A word must be said at this point relative to the function $P(\theta, \Phi)$. Antenna engineers will recognize this function immediately as the so-called "array characteristic." It is that quantity by which the field of a single antenna must be multiplied to give the field of the entire array (assuming like elements). For the two-antenna case, the value of $P(\theta, \Phi)$ when the relative phase of the antennas, and the current amplitudes are adjusted in any arbitrary manner, is

$$P(\theta, \Phi) = \sqrt{1 + K^2 + 2K\cos\left(\gamma - \frac{2\pi d}{\lambda}\cos\Phi\sin\theta\right)}. \tag{8}$$

Here γ is the electrical phase angle between the currents in the two antennas. The sign of γ is to be chosen as indicated in Fig. 1.

Φ is the azimuth angle, measured as shown.

d is the spacing between the antennas in meters (if λ is measured in meters).

K is the ratio of current amplitude in one of the antennas, to the current amplitude in the reference antenna. The amplitude of the latter current is taken as unity.

When $\theta = \pi/2$, (8) becomes

$$P(\Phi) = \sqrt{1 + K^2 + 2K\cos\left(\gamma - \frac{2\pi d}{\lambda}\cos\Phi\right)}. \tag{9}$$

A plot of (9) gives the azimuth pattern in the perfectly conducting plane.

Substituting (8) and (1) in (4),

$$K_0^2(\theta, \Phi) = 4H_e^2\sin^2\theta\left[1 + K^2 + 2K\cos\left(\gamma - \frac{2\pi d}{\lambda}\cos\Phi\sin\theta\right)\right]. \tag{10}$$

Using (5), the radiation is

$$R_0^e = \frac{15}{\pi}H_e^2\int_0^{2\pi}\int_0^{\pi}\left[1 + K^2 + 2K\cos\left(\gamma - \frac{2\pi d}{\lambda}\cos\Phi\sin\theta\right)\right]\sin^3\theta d\theta d\Phi \tag{11}$$

where

$$H_e = \frac{2[J_0(H) - \cos H]}{\sin H}$$

as before.

Several of the terms involved in an expansion of (11) are very simple, and when these are integrated, one may write

$$R_0^e = \frac{15}{\pi}H_e^2\left[(1 + K^2)\frac{8\pi}{3} + 2K\left(\cos\gamma\int_0^{2\pi}\int_0^{\pi}\cos[(\beta_0 d)\cos\Phi\sin\theta]\sin^3\theta d\theta d\Phi + \sin\gamma\int_0^{2\pi}\int_0^{\pi}\sin(\beta_0 d\cos\Phi\sin\theta)\sin^3\theta d\theta d\Phi\right)\right] \tag{12}$$

where β_0 is written for $2\pi/\lambda$.

In (12) one is principally concerned with the evaluation of the integrals. The first one may be written

$$\int_0^{2\pi}\int_0^{\pi}\cos(\beta_0 d\cos\Phi\sin\theta)\sin^3\theta d\theta d\Phi = \int_0^{\pi}\sin^3\theta d\theta\int_0^{2\pi}\cos(\beta_0 d\cos\Phi\sin\theta)d\Phi = \int_0^{\pi}\sin^3\theta d\theta\int_0^{2\pi}\cos(F\cos\Phi)d\Phi \tag{13}$$

where $F = \beta_0 d\sin\theta$ and is constant in the integration of the second factor.

It is known that[18,19]

$$\int_0^{2\pi}\cos(F\cos\Phi)d\Phi = 2\pi J_0(F). \tag{14}$$

[15] E. Hallén, "Theoretical investigations into the transmitting and receiving qualities of antennas," *Nova Acta Upsaliensis*, ser. IV, vol. 11, pp. 1–44; November 11, 1938.

[16] R. King and G. H. Blake, "The self-impedance of a symmetrical antenna," Proc. I.R.E., vol. 30, pp. 335–349; July, 1942.

[17] See page 1802 of footnote 4.

[18] G. N. Watson, "A treatise on the theory of Bessel functions," Cambridge University Press, Cambridge, England, 1922.

[19] N. W. McLachlan, "Bessel Functions for Engineers," Oxford University Press, New York, N. Y., 1934.

Accordingly, (13) becomes

$$2\pi \int_0^{\pi} J_0(\beta_0 d \sin\theta) \sin^3\theta d\theta. \tag{15}$$

Now

$$J_0(Z) = \sum_{r=0}^{r=\infty} (-1)^r \frac{(\frac{1}{2}Z)^{2r}}{(r!)^2}$$

$$= 1 - \frac{Z^2}{2^2} + \frac{Z^4}{2^2 \cdot 4^2} - \frac{Z^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots \tag{16}$$

Also, it can be shown that it is a legitimate procedure to integrate (15) using (16) term by term, as the series is absolutely convergent. Hence

$$2\pi \int_0^{\pi} J_0(\beta_0 d \sin\theta) \sin^3\theta d\theta$$

$$= 2\pi \left[\int_0^{\pi} \sin^3\theta d\theta - \frac{(\beta_0 d)^2}{2^2} \int_0^{\pi} \sin^5\theta d\theta \right.$$

$$\left. + \frac{(\beta_0 d)^4}{2^2 \cdot 4^2} \int_0^{\pi} \sin^7\theta d\theta - \cdots \right]. \tag{17}$$

For convenience in the integration, the formula

$$\int_0^{\pi} \sin^n\theta d\theta = 2\left[\frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{1 \cdot 3 \cdot 5 \cdots n}\right] \text{ for } n \text{ odd} \tag{18}$$

is of use. Using (18) in conjunction with (17), one obtains

$$2\pi \int_0^{\pi} J_0(\beta_0 d \sin\theta) \sin^3\theta d\theta$$

$$= 4\pi \sum_{m=0}^{m=\infty} (-1)^m (\beta_0 d)^{2m} \frac{(2m+2)^2}{(2m+3)!}. \tag{19}$$

A process similar to that used in evaluating (13) may be used in solving the second integral in (12). It turns out that for this case[19]

$$\int_0^{2\pi} \sin(F \cos\Phi) d\Phi = 0 \tag{20}$$

occurs as a multiplying factor, and thus the second integral in (12) vanishes.

The radiation resistance is, therefore,

$$R_0^e = \frac{60}{\pi} \frac{(J_0(H) - \cos H)^2}{\sin^2 H} \left[\frac{8\pi}{3}(1 + K^2) + 8\pi K \cos\gamma \sum_{m=0}^{m=\infty} (-1)^m \frac{(\beta_0 d)^{2m}(2m+2)^2}{(2m+3)!}\right]$$

$$= \frac{15}{\pi}\left[\frac{2(J_0(H) - \cos H)}{\sin H}\right]^2 \left[\frac{8\pi}{3}(1 + K^2) + 8\pi K \cos\gamma \left(\frac{2}{3} - \frac{2}{15}(\beta_0 d)^2 + \frac{(\beta_0 d)^4}{140} - \cdots\right)\right]. \tag{21}$$

An investigation of (21) reveals that for the average antenna spacings, only a few terms in the expansion are required, as the series converges rapidly. Thus the problem of radiation-resistance determination for the two-vertical-antenna array is an extremely simple process, and can be carried out easily by the radio-station personnel.

The directivity of the array is

$$D = \frac{60 H_e^2 [1 + K^2 + 2K]}{R_0^e} \tag{22}$$

if $\beta_0 d > \gamma$. R_0^e is given by (21); H_e by (1). D_{max} occurs at

$$\gamma - \beta_0 d \cos\Phi \sin\theta = 0 \tag{23}$$

or since $\theta = 90$ degrees on the earth's surface,

$$\cos\Phi_m = \frac{\gamma}{\beta_0 d}. \tag{24}$$

Now if $\beta_0 d < \gamma$, $\Phi_M = 0$ degrees if γ is positive and less than 180 degrees. In the latter case,

$$D = \frac{60 H_e^2 [1 + K^2 + 2K \cos(\gamma - \beta_0 d)]}{R_0^e}, \tag{25}$$

which is to say that D_{max} occurs on the line joining the two antennas, and on the same side as the antenna having the lagging current. The angle of lag must not be more than 180 degrees, if the above conditions are to apply.

Upon finding D from either (22) or (25), as the case may be, (7) may be employed directly to obtain the gain in decibels of the array referred to any arbitrary reference antenna.

Example:

Let it be required to investigate the performance of the following array:

Operating wavelength $\lambda = 508.5$ meters
Tower height $h = 106.7$ meters $= 0.21\lambda$
Separation $d = 225.5$ meters $= 0.443\lambda$
Current in east antenna $I_E = 14.2$ amperes
Current in west antenna $I_W = 8.7$ amperes

Here the currents I_E and I_W correspond to the currents I_A and I_B, respectively, in Fig. 1.

The current I_W leads I_E by $\gamma = 30$ degrees.

It is the understanding of the writer that the above data apply to radio station WEEI in Boston, Massachusetts.

It is evident from the data that

$$\beta_0 h = 75.6 \text{ degrees} = 1.32 \text{ radians}$$

and

$$\beta_0 d = 159.6 \text{ degrees} = 2.786 \text{ radians}.$$

If one refers the radiation resistance to the input current in the east antenna, the factor

$$K = 8.7/14.2 = 0.613.$$

Equation (9) then becomes

$$P(\Phi) = \sqrt{1.375 + 1.225 \cos(30° - 159.6° \cos\Phi)}.$$

A plot of this relation is the azimuth directivity of the array in the perfectly conducting plane. The azimuth pattern for this particular antenna is hardly of sufficient interest to warrant reproduction here.

The radiation resistance should be computed from (21). One has

$$\left[\frac{2(J_0(H) - \cos H)}{\sin H}\right]^2 = 0.556.$$

This value may be computed directly or read from the curve shown in Fig. 2, for $H = 1.32$ radians.

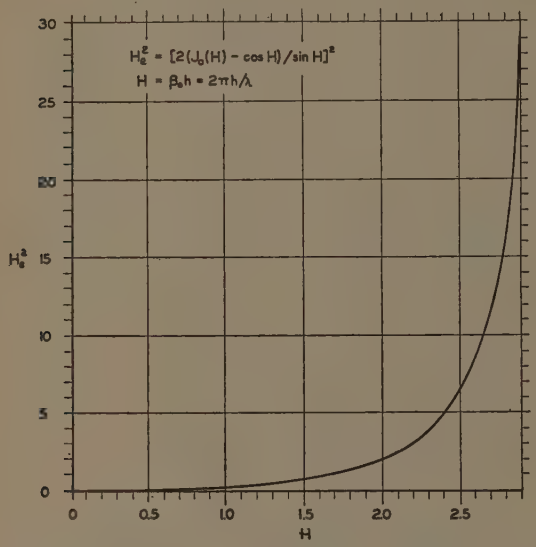

Fig. 2—The square of H_e as a function of the angular antenna length H.

Also, $15/\pi(0.556) = 2.65$. Using the above values, and noting that $\beta_0 d = 2.78$ radians, and $K = 0.613$, (21) becomes

$$R_0^e = 2.65[11.52 + 6.66(1.33 - 2.07 + 0.861 - 0.165 + 0.018 - 0.00013) + \cdots]$$

from which $R_0^e = 30.2$ ohms.

The total power radiated

$$P_0 = I_E^2 R_0^e = (14.2)^2(30.2) = 6.08 \text{ kilowatts.}$$

This is naturally an optimistic result, as ground losses have not been taken into account. A previous set of antenna-current values for this same station were $I_E = 12.4$ amperes and $I_W = 8.6$ amperes. For this case, $R_0^e = 32.5$ ohms referred to I_E, and the power radiated is 5.0 kilowatts. Here again, ground losses have not been considered.

Using (22)

$$D = \frac{60(0.556)[1 + K^2 + 2K]}{R_0^e}$$

where

$$K = 0.613 \quad \text{and} \quad R_0^e = 30.2 \text{ ohms.}$$

The expression for D given above is the correct one, for in this case

$$\cos (30 \text{ degrees} - 159.6 \text{ degrees} \cos \Phi) = 1,$$

when

$$\Phi = 79.2 \text{ degrees.}$$

Hence

$$D = 2.87.$$

Using (7), we find[20] that

$$\text{gain} = 10 \log_{10}\left(\frac{2.87}{1.64}\right) = 2.43 \text{ decibels.}$$

The latter figure represents the gain of the WEEI antenna above a half-wave dipole located in empty space. In this particular instance, the figure for the antenna gain also approximates the gain of the array over a single tower element.

From the general solution of the problem of two vertical antennas, and from the illustrative example, it is clear that the approximate representation of the field characteristic by the leading term of a Fourier expansion is of extreme importance, when one is faced with the solving of intricate antenna problems.

In conclusion, it is well to state that whereas the assumption of a perfect earth was made in carrying out the analysis, a consideration of the effective dielectric constant and conductivity of average soil, as it affects the results, shows that the method of images is satisfactory, when the earth is treated as a perfect conductor for frequencies in the broadcast band, and for longer wavelengths.

Acknowledgment

It is hardly necessary to say that without the prior work of Dr. Ronold W. P. King of the Harvard faculty the writing of this paper would have been impossible.

The author is also indebted to Mr. W. D. Woo, Instructor of Physics and Communication Engineering at Harvard, for his helpful suggestions and check of the mathematical development.

[20] In the special case of a single antenna, the radiation resistance is

$$R_0^s = 15\int_0^{\pi} K_0^3(\theta) \sin \theta d\theta$$

and

$$D = \frac{30K_0^2(\theta)}{R_0^e}.$$

For a half-wave dipole, $R_0^s = 71.44$ ohms and $D_0 = 1.5$. Using this value of D_0, the gain of the WEEI antenna becomes 2.81 decibels.

Lightning Striking Frequencies for Various Heights*

DATA have now been obtained in the lightning investigations conducted by the Westinghouse Company on the number of times per year objects of varying heights are struck in regions of isoceraunic levels varying from 25 to 45 storm days per year. These are shown in Fig. 1 together with similar data which have been obtained on the Empire State Building.

TABLE I

RECORD OF THE NUMBER OF TIMES OBJECTS OF VARYING HEIGHTS ARE STRUCK

Object and Location	Height Feet	Number of Years	Times Struck	Average No. Year
Mast at North Wales Substation (Philadelphia) of Philadelphia Electric Co.	80	4	1	0.25
10 Fire Towers of the Pennsylvania State Department of Forests and Waters, Western Pennsylvania	100	1	2	0.2
Radio Tower of WWSW, Pittsburgh	100	3	1	0.33
Radio Tower of WHK	300	1	1	1.0
Radio Tower of WCLE (Both Cleveland)	300	1	0	0
Radio Tower of WADC	360	3	6	2.0
Cathedral of Learning of University of Pittsburgh	535	3	6	2.7
Anaconda Copper Company Smokestack at Great Falls, Montana	545	2	1	0.5
Anaconda Copper Company Smokestack at Anaconda, Montana	565	2	5	2.5
Empire State Building, New York City	1250	3	68	23

These objects are in regions of isoceraunic levels varying from 25 to 45 storm days per year.

The curve of Fig. 1 was obtained by grouping the data into mean values of height and averaging the strokes per year for each group. A mast 200 feet high can be expected to be struck about once every one and one-half years and a 100-foot mast about once every three years. Laboratory tests indicate that for strokes that do not have appreciable upward leaders, the strokes attracted to a mast increase linearly with the height of the mast. This relation is indicated by the general shape of the lower part of Fig. 1. The upward trend of the curve for high objects is probably due to the upward streamers that occur in nature from objects of such height.

When considering a single building, .1 per cent exposure results in one stroke every 200 to 400 years and 0.1 per cent exposure in one stroke every 2000 to 4000 years. However, many systems have a large number of substations, which increases the over-all exposure. Balanced against the desirability of perfect shielding is the increase in cost incident to taller shielding structures. Certainly not over 1 per cent exposure should be permitted and when a comparison between the height of the shielding structure required to obtain 0.1 per cent exposure over that for 1 per cent exposure is made it will be seen that, in general, the added height can be obtained with little increase in cost. For this reason an exposure figure of 0.1 per cent is used in discussing the shielding of structures.

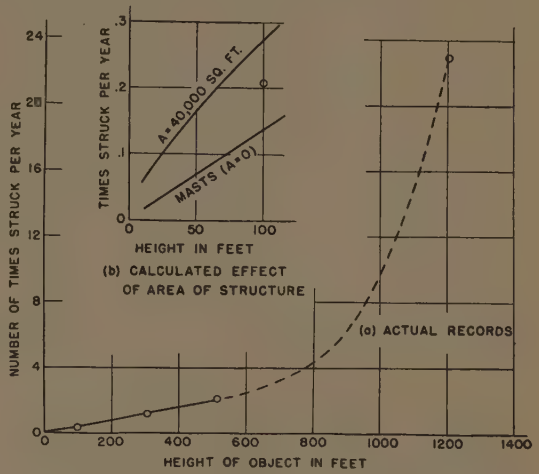
NUMBER OF TIMES STRUCK PER YEAR
TIMES STRUCK PER YEAR
A=40,000 SQ. FT.
MASTS (A=0)
HEIGHT IN FEET
(b) CALCULATED EFFECT OF AREA OF STRUCTURE
(a) ACTUAL RECORDS
HEIGHT OF OBJECT IN FEET

Fig. 1—A chart showing the lightning striking frequencies for varying heights (b) shows the calculated effect of the area of a structure at various heights.

* Decimal classification: 537.4. A release from the Westinghouse Editorial Service, East Pittsburgh, Pennsylvania. Received by the Institute, December 2, 1942.

Discussion on

"A Contribution to the Theory of Network Synthesis"*

R. A. Whiteman

E. A. Guillemin[1]: The contribution of this paper is not evident inasmuch as the central formula, equation (16), is nothing more than the essence of the Fourier-integral method of formulating transient response and hence is as old as this theory itself. It merely expresses the well-known fact that the transform of the response is equal to the product of the transform of the applied signal and the system function.

The heuristic derivation of the Fourier integral given in this paper is likewise well known since the demonstration that the Fourier series is a special case of the Laurent expansion may be found in various textbooks on function theory.

In the belief that equation (16) alone furthers the solution to the general synthesis problem, the author is apparently unaware of the real difficulties of this problem, and the limitations within which this formula can usefully be applied. In the design of two-terminal-pair networks, for example, the formula yields only the transfer function, which alone does not specify the network. The author dismisses this item with the unhelpful remark that "the four-terminal-network problem requires, for its solution, supplementary data in the form of prescribed driving-point-impedance functions." The restrictions on the use of the formula which are imposed by conditions of physical realization are not mentioned.

In the illustrative example (which incidentally is the only novel item in the paper) the author uses the term "equivalent network" in a sense which is decidedly restricted. Unless this restriction is clearly appreciated by the reader, the implication contained in the result may be seriously misleading. Finally, the process of finding a network which realizes the impedance function, equation (20), does not require Brune's method since it is recognized as the impedance of an RC network and hence synthesized by the simpler method of Cauer's extension of Foster's reactance theorem.

R. A. Whiteman[2]: With reference to Professor E. A. Guillemin's letter concerning the above paper, I would like to emphasize a few facts.

It is well known that an important advancement in network synthesis was achieved when a procedure was developed of finding a two-terminal network consisting of the elements R, L, and C that satisfied a given impedance function. For such a procedure to have practical value, however, one or several methods of obtaining $Z(p)$ must be available from prescribed attenuation and phase-shift requirements or their equivalent. The method discussed in the above paper was based entirely upon prescribed transient conditions and, to my knowledge, has not been published in the past.

Equation (16) could have been obtained by other methods, even those used in operational calculus; but the method used was selected because equations (3) and (4) are dependent directly upon the Laurent expansion theorem and not upon the Fourier series. Furthermore, they are not printed in the well-known texts on functions of a complex variable.

The physical realization of a network satisfying the prescribed transient conditions may be determined by applying methods already published concerning the restrictions applied to $Z(p)$.

The illustrative example was merely selected to illustrate the method of using equation (16) and not for the purpose of demonstrating Brune's method. No further justification of the example seems necessary.

E. A. Guillemin[1]: Mr. Whiteman states that his formula, equation (16), has to his knowledge not been published previously. Since the relationship expressed by this equation is the very essence of the Fourier-integral method of transient network analysis and hence familiar to all who have a basic knowledge of the fundamental reasoning upon which this method operates, it seems altogether superfluous to consider it necessary that mention be made of prior publication regarding this item. Indeed, it may prove difficult to determine just when and where this matter was first pointed out. Therefore, I merely call attention to one well-known reference where this formula appears, namely in the Bell System Monograph B-584 entitled "Fourier Integrals for Practical Applications," by G. A. Campbell and R. M. Foster. On page 24, the third of three unnumbered equations expresses (in a different notation, of course) precisely the same relationship as Mr. Whiteman's equation (16). The paragraph immediately preceding the three equations just referred to contains, in a nutshell, the point at issue.

"For any system where the principle of superposition holds, any cause $C(t)$, its effect $E(t)$ and the corresponding admittance $Y(f)$ are connected by a relation which may be written in any one of three ways which explicitly express each of the three quantities in terms of the remaining two. . . ."

I might add that the underlying thought involved here, namely, that the transient behavior of a network is *uniquely* characterized by the steady-state response function of the system, is now so widely used, and is considered of such fundamental importance in many

* Proc. I.R.E., vol. 30, pp. 244–247; May, 1942.

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[2] 6600 N. Bosworth Ave., Chicago, Illinois.

branches of engineering work that the precise equation referred to above has found its way into the regular instruction given in the junior year by the Mathematics Department here at the Massachusetts Institute of Technology.

Regarding the derivation of the Fourier integral from the Laurent expansion I would like to emphasize again that what Mr. Whiteman does in his paper is entirely heuristic and can in no sense be regarded as a mathematical derivation. The usual procedure here (and this is what I refer to in my first letter as being well known) is first to obtain the Fourier series from the Laurent expansion and then proceed heuristically from the Fourier series to the Fourier integral. The first of these steps appears in various places in the literature, for example, in Titchmarch's "Theory of Functions," Oxford Press, 1932, on page 401, and again in Franklin's "A Treatise on Advanced Calculus," John Wiley and Sons, 1940, as problem 13, on page 507. The existence of the rigorous demonstration for the correctness of the heuristic step from the Fourier series to the Fourier integral is so well known that references are unnecessary.

The only novelty about Mr. Whiteman's procedure so far as I am aware is that he makes the heuristic step from the Laurent series to an analogous integral first, and the usual substitution involved in going from the Laurent series to the Fourier series afterward. This interchange in the order of the two essential steps involved is too trivial to be regarded as a contribution. In fact, if it were not for the existence of extensive literature justifying the usual procedure, Mr. Whiteman's "derivation" would have no sound mathematical foundation at all.

The illustrative example given by Mr. Whiteman is interesting but it certainly is not representative of a typical example in network synthesis as one is justified to expect after reading the title to his paper.

Quite apart from the question of whether the presentation in Mr. Whiteman's paper is original or not, it should be observed that the utility of equation (16), in network synthesis according to Brune's method (or other methods proceeding from the same standpoint) is limited because these methods require that the impedance function be given as a quotient of polynomials. The Fourier-integral formulation does not yield the impedance in this form, but must in general be supplemented by an algebraic-approximation process before the usual synthesis procedures can be applied. This is the chief limitation to the utility of the formula in question and is well recognized by those familiar with the art.

R. A. Whiteman[2]: The Bell System Monograph B-584, to which Professor Guillemin refers, is well known and is a standard reference in many engineering schools as well as at the Massachusetts Institute of Technology, although the applications of this material are generally restricted to circuit-analysis problems. Again I mention that to my knowledge this application to network synthesis has not been previously published.

The method used in order to derive equations (3) and (4) was brief and would appear to be a heuristic derivation upon a superficial reading of the paper. The problem of passing from an infinite series to an infinite integral is by no means trivial or superfluous to anyone familiar with Maclaurin's work on real series. The procedure is mathematically straightforward yielding results which are not ambiguous. To illustrate that the method has mathematical foundations, consider the following extensions of Maclaurin's work on real variables.

I. If (a) $f_n(x)$ is convergent and continuous in the interval (a,b) as n continuously approaches infinity

(b) $$S_1 = \sum_{n=0}^{n\to\infty} f_n(x)$$

(c) $$\frac{\partial f}{\partial n} \text{ is continuous and negative}$$

then

$$S_2 = \int_0^\infty f_n(x)dn$$

is finite and

$$S_2 < S_1.$$

II. If (a) $f_n(x)$ is convergent and continuous in the interval (a,b) as n continuously approaches infinity

(b) $$l_1 = \sum_{n=0}^{n\to\infty} f_n(x)$$

then

$$l_2 = \int_0^\infty f_n(x)dn$$

is finite.

III. If $f_n(Z)$ is a complex function of Z, absolutely convergent in the region R and (a) $f_n(Z)$ be a continuous function of n in this region then

$$L_1 = \int_0^\infty |\, f_n(Z)\,|\, dn$$

is finite and

$$L_2 = \int_0^\infty f_n(Z)dn$$

is finite with

$$L_2 \leqq L_1.$$

To apply the above method to the Laurent expansion theorem, first consider (1) and (2) as

$$f(\lambda) = \sum_{f=-\infty}^{f=+\infty} A(f)(\lambda - \lambda_0)^f \cdots \qquad (1)$$

where

$$A(f) = \frac{1}{2\pi j}\oint \frac{f(w)dw}{(w-\lambda_0)^{f+1}} \cdots \quad (2)$$

From Cauchy's integral theorem and the gamma function

$$A(f) = \frac{f^f(\lambda_0)}{\Gamma(f+1)} \cdots \quad (3)$$

which also applies to fractional-order derivatives. Since

$$\lim_{f\to\infty} \frac{f^f(\lambda_0)}{\Gamma(f+1)}(\lambda-\lambda_0)^f \to 0$$

the summation (1) becomes an integral of finite value $\phi(\lambda)$. That is,

$$\phi(\lambda) = \int_{-\infty}^{+\infty} \frac{\phi^f(\lambda_0)}{\Gamma(f+1)}(\lambda-\lambda_0)^f df \quad (4)$$

with

$$\frac{\phi^f(\lambda_0)}{\Gamma(f+1)} = \frac{1}{2\pi j}\oint \frac{\phi(w)}{(w-\lambda_0)^{f+1}} dw \cdots . \quad (5)$$

The quantity $B(f)$ in the original paper is given by the relation

$$B(f) = \frac{\phi^f(\lambda_0)}{\Gamma(f+1)}$$

which correlates (3) and (4) of the original paper with (4) and (5) of this note.

After all of the above correspondence has been read, there does not seem to be any evidence of a similar paper having been published.

Institute News and Radio Notes

Engineers in Wartime

Radio-and-electronic engineers may justly feel proud of their present role. We are pleased to present some encouraging statements made late in December, 1942, by James G. Harbord, Lieutenant-General United States Army (Retired) and Chairman of the Board of the Radio Corporation of America. General Harbord said, in part:

"The United Nations should look forward to 1943 as a year bright with promise in the war against the Axis. Here in the United States, after long, hard months of preparation, we are getting results scarce believed possible a year ago. Millions of men are being equipped and trained in modern warfare. Our industrial capacity has been geared to a speed that will eventually overwhelm the enemy with its weight and power. With all its implications for final victory, this power should come into full force during 1943.

"Real fighting is ahead. Wherever the battle lines are drawn, radio will be in the thick of the fight, for it is the life line of wartime communications on land, sea, and in the air.

"The war map today reveals that American soldiers, sailors, and marines are lined up at more than sixty places on the world-wide fighting front. To unify them in communications is a mighty task. Without radio it would be a slow, almost impossible task. Every outpost, whether in jungles or on glaciers, no matter how remote, is linked to headquarters. American fighting men, almost a million of them, are focused in action by radio—the global life line of communications.

"In World War I, the center of action lay in France. From that battle front radiated the communication lines. Wireless was being given its first wartime test, but at no time did the demands upon it remotely approach those of World War II. In the intervening years, the development of the electron tube, of short waves, and of many other devices and services of radio have tremendously increased the efficiency of communications. The result has been that in 1942, radio was ready to play the vital role assigned to it on the many far-flung fronts. . . .

"Science, through development of the electron tube, put radio in the fight and made it indispensable to the modern mechanized army, to the air corps, to the fleet, and to the merchant marine. Without the radio tube so wonderfully developed since World War I, radio could not play the important role it now has in warfare. The electron tube made radio equipment compact, portable, mobile, efficient, and extremely dependable. That was not so with the cumbersome wireless apparatus that used the spark transmitter and crystal detectors in the first World War. It was not until the final period of the conflict that the radio tube began to find service in the Army and Navy.

"Radio now qualifies as the voice and ear of the Army Signal Corps, of Naval Communications, and of the Air Corps. We have but to look at the global-war map to realize the great importance of radio. Its definite assignments and achievements necessarily are military secrets. But when we compare the present demands upon communications with those of the first World War, it is easy to understand that radio's present role is a thousandfold more important. The airplane, the world-wide transport problem, and blitz warfare, all of which call for utmost speed and efficiency in communication, have multiplied the demands and responsibilities of radio.

"Within the past year—a year of tireless effort in the manufacturing plants—the men and women on the production front have given the American armed forces the finest radio equipment in the world. As the war rages into 1943, every American finds himself and herself linked in some way with the battle. There must be no letup on the home front. Every day in the New Year must find production rushing full speed ahead to the battle fronts. Then, and only then, will the last battle end in our victory."

Executive Committee

The meeting of the Executive Committee, held on December 29, 1942, was attended by Haraden Pratt, acting chairman; I. S. Coggeshall; Alfred N. Goldsmith, editor; R. A. Heising (guest); F. B. Llewellyn; B. J. Thompson; and W. B. Cowilich, assistant secretary.

Approval was granted to 80 applications for Associate, 117 for Student, and 2 for Junior grades.

The firm of certified public accountants, to make the financial audit of the Institute's 1942 records, was chosen.

Miss Martha Stevens has been employed to fill the vacancy of office manager. As a result of changes in staff and editorial procedure, Mrs. Peggy Nelson has been engaged as editorial assistant.

Editor Goldsmith reported on the number of papers received for the PROCEEDINGS and on the procedure and prospects for obtaining others in the future.

A further discussion was held on the subject of the budget for 1943. Consideration was given to adjustment of salaries of the office staff.

The Executive Committee met on January 4, 1943, and those in attendance were Haraden Pratt, acting chairman; I. S. Coggeshall (guest); Alfred N. Goldsmith, editor; R. A. Heising (guest); F. B. Llewellyn (guest); B. J. Thompson, L. P. Wheeler (president-elect); and W. B. Cowilich, assistant secretary.

The budget for 1943, with further revisions, was approved for submission to the Board of Directors for affirmative action.

The Institute's contract with Mr. W. C. Copp, responsible for soliciting advertising in the PROCEEDINGS, was amended with regard to the rate of compensation paid Mr. Copp, subject to the approval of the Board of Directors.

The personnel of several committees was developed for recommendation to the Board of Directors.

Assistant Secretary Cowilich reported that the Victory Tax, applying to the salaries of the entire staff, is being put into effect.

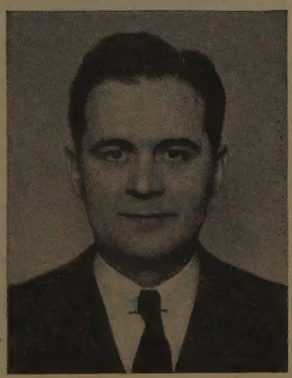
Mr. Pratt presented information on the recent meeting of the Consultative Committee on Engineering of the Division of War Manpower Commission, which he attended as an alternate for President Van Dyck, prior to submitting a similar report to the Board of Directors.

Mr. Thompson reported on his investigation of the subject of draft deferment of radio personnel. President-elect Wheeler was appointed chairman of a committee on the subject, with authority to select the personnel of the particular committee, and it was agreed to recommend this action to the Board of Directors.

Editor Goldsmith called attention to the possible extension of censorship requirements applying to mailing of PROCEEDINGS to countries other than those now thus affected. Recommendation was made to the Board of Directors that a suitable and specific policy be established.

Editor Goldsmith also reported on the progress of the standards work being done by the Sectional Committee on Radio of the American Standards Association, of which he is chairman. It was learned that Editor Goldsmith, acting as Institute representative through A.S.A., is active in work for the War Production Board in standardization of ceramics for radio insulations.

Contributors



A. B. BERESKIN

Alexander B. Bereskin (A'41) was born in San Francisco, California, on November 15, 1912. He received the E.E. degree

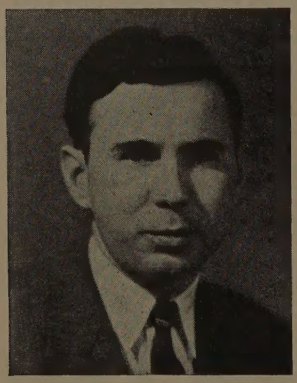
from the University of Cincinnati in 1935. This was followed by work with the Commonwealth Manufacturing Corporation from 1935 to 1937 and with the Cincinnati

C. W. HARRISON, JR.

Gas and Electric Company from 1937 to 1939. In 1939 he returned to the University of Cincinnati as a teaching Fellow and received the M.Sc. degree in engineering in 1941. Since that time Mr. Bereskin has been connected with the electrical engineering department of the University of Cincinnati as an instructor. He is a member of Sigma Xi and the A.I.E.E.

❖

Charles W. Harrison, Jr. (A'36) was born in Virginia in 1914. He has been a student at the Naval Academy, University of Virginia, Harvard University, and the Massachusetts Institute of Technology. His experience includes amateur, naval, and broadcast-station operation, as well as research work at the Navy Department in Washington. At the present time Mr. Harrison is a member of the Cruft Laboratory Staff, Harvard University.



GEORGE F. LEVY

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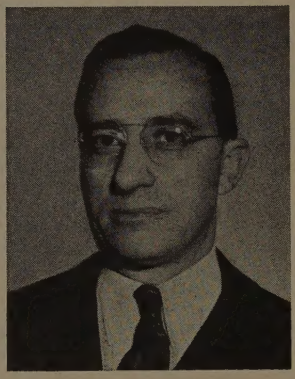
George F. Levy (S'37-A'39) was born in Chicago, Illinois, on July 11, 1913. He received the B.S. degree in electrical engineering from Purdue University in 1937. From 1937 to date he took postgraduate work in electrical engineering at the Evening Graduate School of the Armour Institute of Technology, now the Illinois Institute of Technology. In 1933 and 1934 he was employed in the test and inspection department of the Grigsby-Grunow Manufacturing Company and in 1934 and 1935 he worked in the engineering department of the Zenith Radio Corporation. During



H. E. ROYS

1942 Mr. Levy was a part-time instructor in radio at the Northwestern Technological Institute. Since 1937 he has worked as a communication engineer in the communications laboratory of the United Air Lines Transport Corporation.

❖

H. Edward Roys (A'27) was born at Beaver Falls, Pennsylvania, on January 7, 1902. He received the B.S. degree in electrical engineering from the University of Colorado in 1925. He entered the radio test section of the General Electric Company in 1925, and was transferred to the RCA Manufacturing Company in 1930 as a development engineer. He is still with the same company, and his work has been mainly with disk reproducers, recorders, and turntable drivers. Mr. Roys is a member of Eta Kappa Nu and Tau Beta Pi.

❖

William G. Shepherd (A'42) was born on August 28, 1911, at Fort William, Ont., Canada. He received the B.E.E. degree in 1933 and the Ph.D. degree in physics in 1937 from the University of Minnesota. From 1933 to 1937, Mr. Shepherd was a teaching fellow in physics at the University of Minnesota. Since 1937 he has been a member of the research department of the Bell Telephone Laboratories, engaged in

W. G. SHEPHERD

nonlinear circuit and electronics research. He is a member of the American Physical Society and Sigma Xi.

❖

C. L. Townsend* (A'42) was born on July 9, 1909, and began his radio career in 1927. Two years of commercial broadcasting led to a year as sound engineer in Hollywood. For eleven years he has been a member of the National Broadcasting Company's engineering department, the last five of which he has been assigned to television operational development. Since the recent reduction in television service, he has been engaged in development work on NBC contracts with the National Defense Research Council.

* Paper appeared in the January, 1943, issue of the PROCEEDINGS.

C. L. TOWNSEND

Correspondence

The Potentiometer Idea in Network Calculation

In alternating-current circuit work, it frequently happens that the ratio between two complex voltages has to be calculated. In the emergency situation of today it is important that alternating-current calculations be performed as fast as possible and with a minimum chance for error. It seems to the writer that the potentiometer idea of handling networks is unfamiliar to many engineers and students. They start the calculation of a complex voltage ratio by writing an expression for the circuit current, only to eliminate the current before they give the final answer. The disadvantage of this widely adopted method compared with the potentiometer method is twofold:

1. *The calculation takes more time and requires more space.*
2. *The sign and direction of the current must be decided, and if that is not done correctly, an error may result.*

An example of the method where an auxiliary current is introduced is shown on page 503 of the November issue of the PROCEEDINGS. Here, in the interesting article on "The Q Meter and Its Theory," a current is introduced and written one way in equation (1) and another way[1] in equation (2). By substitution, a relation between two voltages V and e is obtained and the current drops out. The circuit is shown in simplified form in Fig. 1a. If, instead of the conventional method, the potentiometer idea is applied, the final equation (4) is easily written down directly as

$$\frac{V}{e} = \frac{1/j\omega C}{R_a + j\omega L_a + 1/j\omega C} = \frac{1}{1 - \omega^2 L_a C + j\omega R_a C}.$$

An RC-coupled amplifier at low frequencies provides another typical example. Suppose the gain of the amplifier shown in Fig. 1b is to be calculated. Following the more or less conventional idea of introducing currents, we may obtain

$$I_p = \frac{\mu E_g}{r_p + Z}$$

$$E_p = -Z_c I_c = -ZI_p$$

$$E_b = R_c I_c$$

$$E_b = \frac{-R_c}{Z_c} E_p = \frac{-R_c}{Z_c}(-Z)\frac{\mu E_g}{r_p + Z}$$

$$V.A. = \frac{E_b}{E_g}$$

$$= \frac{\mu}{\dfrac{r_p(R_c + jX_c)(R_b + R_c + jX_c)}{R_c R_b (R_c + jX_c)} + \dfrac{R_c + jX_c}{R_c}} \quad (1)$$

$$V.A. = \frac{\mu}{1 + \dfrac{r_p}{R_b} + \dfrac{r_p}{R_c} + j\dfrac{X_c}{R_c}\left(1 + \dfrac{r_p}{R_b}\right)} \quad (2)$$

where $X_c = -1/\omega C$. If a sign error enters into the equations involving currents, the answer may come out with a minus sign, representing a phase shift of 180 degrees that does not exist with the notations adopted in Fig. 1b and 1c.

The potentiometer method gives directly

$$E_b = \frac{R_c}{R_c + jX_c} \times \frac{Z}{r_p + Z} \times \mu E_g$$

$$V.A. = \frac{E_b}{E_g} = \text{(same as equation (1))}$$

$V.A.$ = (same as equation (2)).

As no current is involved here, no difficulty with additional signs and their relation to directions in the diagram enter in.

In the article referred to, the voltages are introduced without direction. Although this is a common practice, it is dangerous as soon as the circuits get complicated and ought to be avoided. Further, the calculation is performed in $j\omega L$'s and $1/j\omega C$'s instead of jX's, and as this again is common practice, no criticism of the author of the valuable article on the Q meter is intended. In equation (8) the absolute impedance has the same notation as the complex impedance. Again this is often done in practice, but it is dangerous.

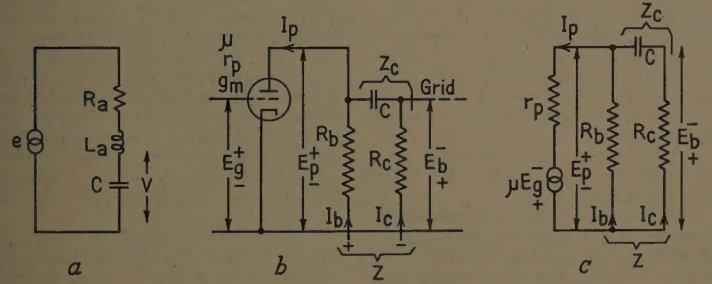


Fig. 1

The points made above along with some others are summarized as follows:

1. Draw circuit diagrams and avoid introducing voltages without putting signs on them. Very often a voltage (or current) notation without indication of direction is meaningless.
2. Do not introduce currents if currents are not asked for or required for any special reason.
3. Make use of the potentiometer idea.
4. If there is not a good reason for the contrary, put everything in formulas of the "Ohm's-law" type with positive sign, and check that the equivalent circuits satisfy conditions by having the voltages aiding the currents.
5. Start the calculation with Z's or Y's and introduce R's and X's or G's and B's when needed. Finally introduce ωL's, ωM's, and $1/\omega C$'s. This means that a choice must be made between $X_c = 1/\omega C$ or $X_c = -1/\omega\ C$. The latter assumption is recommended.
6. Use different notations for complex values and absolute values.
7. Look upon j, $e^{i\omega t}$, $|Z|e^{i\phi}$, etc., not as mathematical symbols, but as turning operators. This is to emphasize the fact that they provide phase shifts when operating upon certain quantities. Thus the impedance operator when operating upon a current will produce a voltage leading the current by the angle ϕ.

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[1] Obviously, C_b in equations (1) to (4) should be C.

Books

Transients in Linear Systems, Volume I, by Murray F. Gardner and John L. Barnes.

Published, 1942, by John Wiley and Sons, 440 Fourth Avenue, New York, N. Y. 382+x pages+7-page index. 178 figures. 6×9 inches. Price, $5.00.

Written primarily for first-year graduate students in electrical and mechanical engineering, *Transients in Linear Systems* should prove of value to any engineer or physicist interested in transients in electrical, mechanical, or electromechanical circuits. The text assumes a working knowledge of differential equations and elementary circuit theory. Additional mathematics is presented as needed in the development of the subject matter.

Volume I treats transients in circuits containing lumped parameters by means of the Laplace transformation. Distributed-parameter circuits are reserved for Volume II which is to be published at a later date. The authors point out the advantages of the Laplace-transform solution over the other three common methods: direct solution of the differential equations, Cauchy-Heaviside operational calculus, and Fourier-integral analysis. The Fourier transformation, it is noted, is a special case of

the Laplace transformation. In many common classes of problems the Laplace transformation is simpler to use than the Fourier transformation. This is the type of problem treated in the text. The Laplace transformation justifies the methods of the operational calculus, but since it is rigorous, the Laplace method has greater possibilities for future development. The Laplace method can be used in any problem where the other methods are applicable, and in many additional problems.

The second chapter of the book is devoted to the setting up of the network differential equations. This material applies equally well for any method of solution, and is excellent review material for the engineer.

An extensive table of transform pairs is built up as the subject is developed, making it unnecessary to introduce the inverse Laplace transformation and integration in the complex plane until the last part of the volume. It is, therefore, not necessary to have a knowledge of complex-plane integration to work through the book. Numerous practical illustrative examples are fully worked out in the text, and practice problems are placed at the ends of each of the nine chapters. Volume I concludes with a chapter on integrodifferential difference equations. Tables of operation-transform pairs and function-transform pairs, and an extensive bibliography including references of historical interest are included in the appendix.

Though the volume includes both electrical and mechanical problems, the engineer interested in only one of these will not find it necessary to go through a large amount of extraneous material, for the two types of systems are considered separately. Most of the material applies equally to the two types of problems.

D. B. Hoisington
Hazeltine Service Corporation
Little Neck, L. I., N. Y.

The "Radio" Handbook, Eighth Edition, 1941

Published by Editors and Engineers, Ltd., 1300 Kenwood Rd., Santa Barbara, Calif. 624 pages+15-page index. 574 figures+7 pages of socket dimensions. 7×10 inches. Price, $1.75.

This book is directed to the serious amateur and experimenter having an elementary knowledge of amateur radio. (For the true beginner, an introductory volume "The 'Radio' Amateur Newcomers' Handbook" is provided.)

The introductory chapter covers the general activities and services of amateur radio, the approach to the study of the code, and of obtaining amateur licenses.

Two chapters on Fundamental Electrical and Radio Theory and Vacuum Tube Theory cover in concise form circuit theory and electronics; electromagnetism is well covered in a presentation paralleling the development of the electric circuit. Brief, but clear, discussions are given of alternating currents, complex impedance, rectification, inductance and mutual inductance; similar ground is covered in electrostatics. The principles of thermionic emission, types of emitters, action of vacuum tubes of varying number of electrodes, space charge, and secondary emission, are covered in the second chapter. Under applications and operation, the use of tubes as amplifiers, with calculation of stage gain, and the various methods of operation are fully covered.

Three chapters on radio receiver theory, receiving-tube characteristics, and receiver construction cover detailed descriptions of the design and operation of elements of receivers and the construction of several types of receivers ranging from a simple two-tube autodyne through superheterodynes. Converters and special high-selectivity intermediate-frequency amplifiers for use with superheterodynes are also described. Wiring diagrams, parts lists, and photographs are included. The list of receiving tubes is very complete and includes obsolescent types. Unfortunately tubes on current preference lists are not specially identified, nor are the several manufacturers identified by name.

The chapter on Transmitter Theory follows the approach developed in the chapters on receiver theory—that of discussing the design and operation of elements of transmitters before going into the construction of the whole transmitter. Such elements include oscillators, crystal-controlled oscillators, oscillator multipliers, amplifiers, neutralized amplifiers, tank-circuit design, grid-bias methods, interstage-coupling units, keying circuits, and so on.

Telephony theory deals similarly with the elements of modulation systems, microphones, speech amplifiers, and inverse-feedback systems in both amplifiers and entire transmitters.

A chapter on Frequency Modulation covers both transmission and reception detailing the elements of systems in phase and frequency modulation, methods of checking linearity, and frequency deviation in transmitters. The part on receivers covers the elements of receivers including discriminators, limiters, and design considerations involving features dependent on bandwidth.

The chapter on transmitting tubes lists characteristics of all usual tubes, with socket connections.

The chapters dealing with transmitter construction are based on the possibility of using a low-powered transmitter, at some subsequent time, as an exciter for a higher-powered amplifier unit. On this basis, six low-powered exciter (or transmitter) units are described, with five suitable medium- and high-powered amplifiers and nine speech-amplifier and modulator units. General considerations of rectification, filtering, regulation, and stabilization of power-supply voltages are followed by constructional data on regulated plate and bias supplies. After the construction of transmitters on a "unit" basis is heated, six different transmitters are described, to be built as complete assemblies. These include both phone and continuous-wave transmitters with crystal or variable-frequency-oscillator control. The designs described, for both the "unit" and complete assemblies, are clean cut and represent the best in amateur construction.

The ultra-high-frequency field is covered in several chapters, dealing with propagation, receivers and transceivers, and transmitters. Constructional data are given for several receivers and transceivers for use on frequencies up to 400 megacycles, and on numerous transmitters for powers up to 125 watts and frequencies up to 224 megacycles.

Several chapters are devoted to antenna theory and operation. These cover short-wave propagation, the mechanism of radiation, antenna design and characteristics, and methods of feeding antennas; various types of antenna-line matching arrangements are described, and constructional data given. The theory of usefulness of directive arrays in amateur work is given, dealing with both vertical and horizontal directivity. Various types of directive antennas are described, but only those which are practical for amateur use are included. Particular attention is given to ultra-high-frequency antennas with regard to mechanical construction, for both directional and nondirectional types.

A chapter is devoted to Transmitter Adjustment, covering many suggestions and procedures for tuning up a transmitter, which should prove very helpful to the inexperienced.

Under Test and Measuring Equipment, descriptions and constructional data are given for many items, ranging from volt-ohmmeters, wavemeters, and crystal-controlled calibrators to vacuum-tube voltmeters, field-strength meters, signal generators, and cathode-ray oscillographs.

The final chapters cover Workshop Practice, Broadcast Interference, and Radio Mathematics and calculations. The suggestions as to methods of handling work, choice of tools, and layout of workshop space are helpful for those who approach amateur radio without benefit of shop hobbies. Many suggestions are given as to means for avoiding or eliminating interference with broadcast reception, several of which should be very helpful to the amateur plagued by this misfortune. The Radio Mathematics covers logarithms, decibel expression of gain-loss, power levels, and voltage ratios. Reactance-frequency and coil-design charts are included.

The Appendix covers Radio Laws, "Q" Signals, Phillips code abbreviations, artificial respiration, a study guide for amateur class B licenses, advertisements of suppliers, and a complete index.

No attempt has been made to check either formulas or material for accuracy throughout. In passing, the confusion of "affected" and "effected," and the error in condensive reactance (page 380 $\frac{1}{2}\pi fc$ for $1/2\pi fc$) may be noted.

The book covers a wide field in a concise and readable manner. Oversimplification, to meet the knowledge of the amateur has not impaired the accuracy of general concepts. The book should prove valuable not only to the amateur operator and experimenter as a guide, but also to the engineer as a reference of amateur methods and progress.

J. K. Clapp
General Radio Company
Cambridge, Mass.